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MULTI-PERIOD EQUILIBRIUM / NEAR-EQUILIBRIUM IN ELECTRICITY MARKETS BASED ON LOCATIONAL MARGINAL PRICES

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TESIS DOCTORAL

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PhD THESIS

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Ciudad Real, July 2005

A mis padres, por apoyarme y
confiar siempre en mí

A mis hermanos, Javier y Jorge Luis,
por su cariño y sus ánimos

A José Agustín, por todo su amor

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Notation

The main notation used in this thesis is listed below.

Variables

$c^{\mathrm{sd}}_{Gi}(t)$	Shut-down cost of generating unit i in period t.
$c^{\mathrm{su}}_{Gi}(t)$	Start-up cost of generating unit i in period t.
$P_{Djk}(t)$	Power block k that the demand j is consuming in period t.
$\tilde{P}_{Djk}(t)$	Power block k that the demand j is consuming in period t. This variable is equal to $P_{Djk}(t)$ and is used in the problem of the independent system operator.
$P_{Gib}(t)$	Power block b that the generating unit i is producing in period t.
$\tilde{P}_{Gib}(t)$	Power block b that the generating unit i is producing in period t. This variable is equal to $P_{Gib}(t)$ and is used in the problem of the independent system operator.
$P_{nm}(t)$	Real power flow through the line $n-m$ computed at node n in period t.
$P^{\mathrm{loss}}_{nm}(t)$	Real power losses in the line $n-m$ in period t.
u_{1ip}	Variable used to represent the piecewise linear block p of the quadratic term $(x_{iN_{Gi}-1})^2$ of the minimum profit condition of unit i.
u_{2ip}	Variable used to represent the piecewise linear block p of the quadratic term $(x_{iN_{Gi}})^2$ of the minimum profit condition of unit i.
α	Lower bound approximation of the objective function of the equilibrium problem.
$\alpha_{nm,l}(t)$	Slope of the voltage angle difference block l between nodes n and m in period t.

$\delta_n(t)$ Voltage angle of node n in period t.

$\delta_{nm}(t)$ Voltage angle difference between nodes n and m in period t.

$\delta_{nm,l}(t)$ Voltage angle difference block l between nodes n and m in period t.

$\delta^+_{nm,l}(t)$ Positive part of the voltage angle difference block l between nodes n and m in period t.

$\delta^-_{nm,l}(t)$ Negative part of the voltage angle difference block l between nodes n and m in period t.

$\rho_{n(i)}(t)$ Locational marginal price corresponding to the generating unit or the demand i in period t that is located at node n.

Dual variables

$\alpha_i(t)$ Dual variable associated with the maximum capacity constraint of generating unit i in period t.

$\beta_i(t)$ Dual variable associated with the minimum power output constraint of generating unit i in period t.

$\gamma_{nm}(t)$ Dual variable associated with the transmission capacity constraint of line $n - m$ in period t.

$\zeta_n(t)$ Dual variable associated with the upper bound of the voltage angle of node n in period t.

$\zeta^+_{nm,l}(t)$ Dual variable associated with the upper bound of the positive part of the voltage angle difference block l between nodes n and m in period t.

$\zeta^-_{nm,l}(t)$ Dual variable associated with the upper bound of the negative part of the voltage angle difference block l between nodes n and m in period t.

$\eta_{ib}(t)$ Dual variable associated with the nonnegative levels of power constraint for block b of generating unit i in period t.

$\kappa_{vi}(t)$ Dual variable associated with the constraint that fixes the binary variable $v_i(t)$ in the subproblem.

$\mu_{Gib}(t)$ Dual variable associated with the constraint that makes the power generated by block b of unit i in period t equal in the problem of the independent system operator and in the problem of the generating companies.

$\nu_{Djk}(t)$ Dual variable associated with the constraint that makes the power demanded by block k of demand j in period t equal in the problem of the independent system operator and in the problem of the consumers.

$\sigma_j(t)$ Dual variable associated with the minimum demand constraint of demand j in period t.

$\tau_i(t)$ Dual variable associated with the available maximum power output constraint of unit i in period t.

$\phi_{ib}(t)$ Dual variable associated with the maximum capacity limit for block b of generating unit i in period t.

$\varphi_{jk}(t)$ Dual variable associated with the maximum capacity limit for block k of demand j in period t.

$\psi_i(t)$ Dual variable associated with the available minimum power output constraint of unit i in period t.

Binary variables

$v_i(t)$ On / off status of generating unit i in period t (1 if generating unit i is on at hour t and 0 otherwise).

z_{1ip} Binary variable that ensures the sequencing of block p used to linearize the quadratic term $(x_{iN_{Gi}})^2$ of the minimum profit. condition of unit i

z_{2ip} Binary variable that ensures the sequencing of block p used to linearize the quadratic term $(x_{iN_{Gi}-1})^2$ of the minimum profit condition of unit i.

Constants

a Constant ($a \geq 1$).

b Per unit constant ($0 \leq b \leq 1$).

b_{ip} Break point p used to linearize quadratic terms of the minimum profit condition of generating unit i.

B_{nm} Susceptance of the line $n - m$.

C Infeasibility cost.

C_{Gi}^{fx} Fixed cost coefficient of generating unit i.

C_{Gi}^{sd} Constant shut-down cost of generating unit i.

C_{Gi}^{su} Constant start-up cost of generating unit i.

C_i Additional cost assigned to generating unit i due to the infeasibility cost.

C_j Additional cost assigned to demand j due to the infeasibility cost.

CS Consumer surplus.

DSW Declared social welfare.

G_{nm} Conductance of the line $n - m$.

K_i Positive constant that represents the minimum profit imposed by the generating unit i.

M Sufficiently large positive constant.

MS Merchandising surplus.

$P_{Dj}^{\min}(t)$ Minimum power supplied to demand j in period t.

$P_{Djk}^{\max}(t)$ Maximum power demanded in block k of demand j in period t.

$P_{Gi}^{\max}$ Maximum power output of generating unit i.

$P_{Gi}^{\min}$ Minimum power output of generating unit i.

$P_{Gib}^{\max}(t)$ Maximum power output in block b of generating unit i in period t.

$\hat{P}_{Gib}^{(\eta)}(t)$ Estimate of power block b of generating unit i in period t at iteration η.

$P_{nm}^{\max}$ Transmission capacity limit of line $n - m$.

PS Producer surplus.

R_i^{dn} Ramp-down limit of generating unit i.

R_i^{sd} Shut-down ramp limit of generating unit i.

R_i^{su} Start-up ramp limit of generating unit i.

R_i^{up} Ramp-up limit of generating unit i.

SW Social welfare.

Uplift_i	Uplift paid to generating unit i due to the infeasibility cost.
Uplift_j	Uplift paid to demand j due to the infeasibility cost.
$\bar{v}_i(t)$	On / off status of generating unit i in period t fixed in the subproblem to the values obtained in the master problem.
$Z^{(\nu)}_{\text{down}}$	Lower bound of the optimal value of the objective function of the equilibrium problem.
Z_{QPP}	Objective function of the quadratic programming problem that corresponds to the sum of the complementarity conditions of the mixed linear complementarity problem.
$Z^{(\ell)}_{\text{Sub}}$	Objective function of the subproblem at iteration ℓ.
$Z^{(\nu)}_{\text{up}}$	Upper bound of the optimal value of the objective function of the equilibrium problem.
$\alpha^{\min}$	Minimum value for α.
$\Delta\delta$	Piecewise angle block length.
ϵ	Tolerance of the successive over-relaxation iterative algorithm.
ε	Tolerance of Benders decomposition algorithm.
$\lambda^{\text{B}}_{Djk}(t)$	Price bid by demand j to buy power block k in period t.
$\lambda^{\text{U}}_{Djk}(t)$	Marginal utility associated with power block k of demand j in period t.
$\lambda^{\text{B}}_{Gib}(t)$	Price bid by generating unit i to sell power block b in period t.
$\lambda^{\text{C}}_{Gib}(t)$	Linear operating cost of power block b of generating unit i in period t.

Sets

D	Set of indices of demands.
D^{uplift}	Set of indices on the demands that are paid an uplift.
D_q	Set of indices of the demands owned by the consumer q.
G	Set of indices of generating units.
G^{M}	Set of indices of generating units that declare a minimum profit condition.

$G^{\mathrm{M^{on}}}$ Set of indices of generating units that declare a minimum profit condition and remain on-line during at least one time period on the market horizon.

G^{uplift} Set of indices of generating units that are paid an uplift.

G_f Set of indices of generating units owned by the generating company f.

N Set of nodes.

T Set of considered time periods.

Ω_n Set of indices of nodes connected to node n.

θ_n Set of indices of generating units at node n.

ϑ_n Set of indices of demands at node n.

Numbers

L Number of blocks for the linearization of losses.

N_D Number of demands.

N_{Dj} Number of blocks demanded by demand j.

N_{DK} Number of blocks demanded by all demands.

N_G Number of generating units.

$N_{G^{\mathrm{M}}}$ Number of generating units that impose minimum profit conditions.

$N_{G^{\mathrm{M^{on}}}}$ Number of generating units that impose minimum profit conditions and remain on-line during at least one time period on the market horizon.

N_{GB} Number of blocks bid by all generating units.

N_{Gi} Number of blocks bid by generating unit i.

N_L Number of lines.

N_N Number of nodes.

N_T Number of time periods.

P Number of blocks for the linearization of the minimum profit conditions.

η Iteration counter of the successive over-relaxation iterative algorithm.

ν Iteration counter of Benders decomposition algorithm.

Matrices

D Diagonal matrix.

H Hessian matrix.

Q Orthogonal matrix.

Chapter 1

Introduction

1.1 Electricity Markets

Electricity markets are being implemented throughout the world as a result of the restructuring of the electric power industry. Electricity market designs may differ depending on countries and regions but the main purpose of all of them is to regulate energy transactions and their associated economic interchanges. This section describes electricity markets and their operation models.

1.1.1 Towards Restructured Electricity Markets

Traditionally, the supply of electric energy was carried out by a conglomerate of private and vertically integrated utilities. A vertically integrated utility owns and manages generation, transmission and distribution over a certain area. Since there is no competition, such an industry is regulated by the government in order to protect society by controlling investment and tariffs.

In the late 1980s, worldwide power utilities started moving from a traditional monopoly framework to competitive markets becoming horizontally integrated. The main objectives of power system restructuring were to guarantee the electricity supply to all consumers, to guarantee the quality of this supply and to reach the two previous objectives at the minimum cost for the end consumers, [17, 55, 56, 61, 81, 82, 85].

Power system restructuring makes it possible the identification and unbundling of various tasks which were normally carried out within the traditional organization so that these tasks can be open to free competition. Restructuring of the electricity supply industry involves turning generation and retailing into competitive activities, and allowing open access to transmission and distribution grids.

In this scheme, the most commonly electricity market models[56] are: i) a bilateral contract market structure, where generating companies and consumers engage in negotiated contracts to exchange electricity; and ii) a pool-based electricity market structure, where all the entities submit their bids to a pool and an operator clears the market, which results in final energy transactions and prices. This operator uses a clearing procedure that depends on the market model. Practically, all market implementations have one of these two models as their predominant structure but often include some elements of the other model as well. Markets like these of PJM, New Zealand, Australia, Spain and the Nordic power market are predominantly pool-based markets, while bilateral trades predominate in the market of Texas, California, and England and Wales.

1.1.2 Electricity Market Agents

Market agents involved in the generation, transmission, distribution, commercialization and consumption of electric energy in a restructured electricity market are briefly described below [56, 81, 82].

In general, technical and business entities in a competitive electricity market are grouped into generating companies, marketers, consumers, an Independent System Operator (ISO) and a Market Operator (MO).

- **Generating companies**

  Generating companies are entities that own generating units, and operate and maintain them. These companies sell electricity either directly to the consumers, through bilateral contracts, or to a pool. The target of the generating companies is to produce electricity at maximum profit.

- **Consumers**

  Consumers are entities that purchase electricity either from the generating companies, through bilateral contracts, or from the pool in order to supply their respective demands. Consumers seek to maximize their respective economic utilities.

- **Marketers**

  Marketers are entities that buy and sell electricity but do not own generating units, that is, marketers mainly buy electricity from the generating companies and sell it to the consumers.

- **Independent system operator**

  The independent system operator is an entity independent of any agent with commercial interests, and it is responsible for the technical management of the system [17]. It provides open access to the transmission system in a non-discriminatory manner.

  Electricity is by its nature difficult to store and has to be available on demand. Demand and supply vary continuously. Therefore, there is a physical requirement for a controlling entity to continuously maintain the balance of the system. The ISO guarantees the instantaneous balance of the system, electricity supply security, and proper coordination of the production and the transmission systems. It is also responsible for maintaining system reliability and coordinating maintenance scheduling.

- **Market operator**

  The market operator is an entity responsible for the financial management of the system. Mainly, the market operator manages electric energy purchases and sale bids, settles prices and assigns energy to each generating company, consumer and marketer, as well as publishing information regarding market results.

  In some market structures, the independent system operator and the market operator are separate entities. While in others, the market operator is in the same organization as the independent system operator, in which case the independent system operator is responsible for both the economic and technical management of the market.

  The markets of Spain and New Zealand incorporate two separate entities, that is, an independent system operator and a market operator, while in the PJM and California markets the market operator is merged with the ISO, and as a result there is only one operator.

1.1.3 Pool-Based Electricity Market

This thesis focuses on a pool-based electricity market that includes generating companies, consumers and an independent system operator because this market model is widely used in actual markets as stated in Subsection 1.1.1. Participation in the pool is mandatory and there is no direct trade between market agents. We assume that the ISO is responsible for the financial management of the market as well as the management of the transmission network because this is the case of several market structures as can be seen in Subsection 1.1.2, and because is more efficient since the same entity deals with economic and technical management.

In such a pool-based electricity market, the generating companies submit bids to the pool consisting of energy blocks and their corresponding minimum selling prices for every hour of the market horizon and every unit, while the consumers submit energy blocks and their corresponding maximum buying prices for every hour of the market horizon and every demand. Note that sale bids and purchase bids do not necessarily reflect costs and utilities, respectively. The ISO collects purchase and sale bids and clears the market seeking maximum social welfare and using an appropriate market-clearing procedure, which results in hourly prices, and production and consumption schedules. Figure 1.1 shows a representation of a pool-based electricity market.

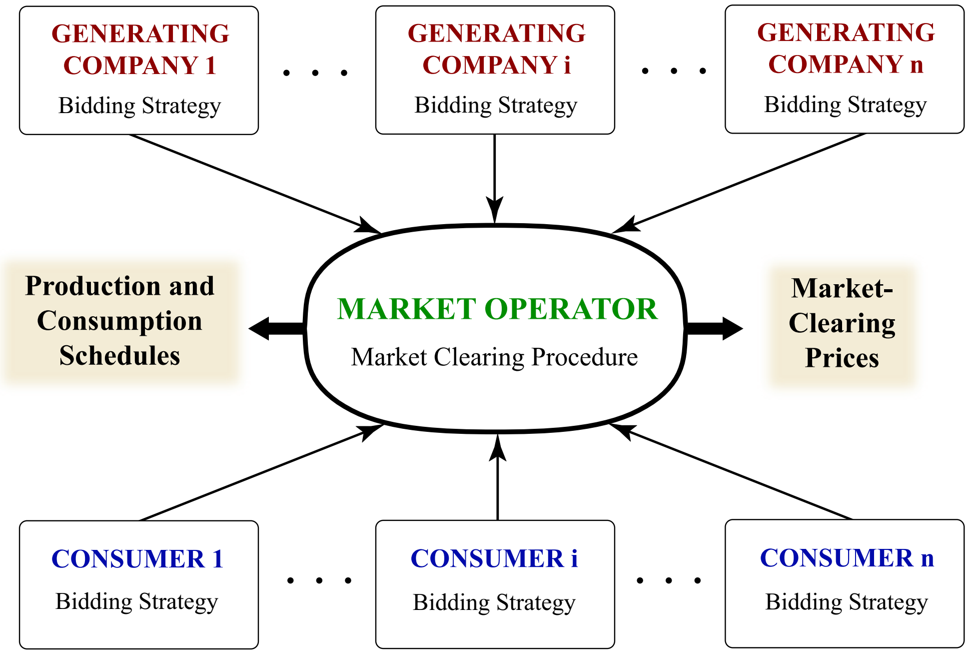

Figure 1.1: Pool-based electricity market

The price of electricity at each node in the network results from the electricity bid prices and the marginal cost of losses and congestion in the network. This nodal price is known as Locational Marginal Price (LMP) [80]. Line losses and congestion can limit the open access of some generating companies or consumers to the transmission network. For example, a line congestion could prevent a generating unit satisfying its scheduled production, and force a more expensive unit to produce an extra energy to compensate this, provoking price differences between nodes where those units are located. In this scheme, a generating unit injecting energy at a given node is paid the locational marginal price corresponding to that node. Conversely, a demand receiving energy from a given node pays the locational marginal price corresponding to that node. Mature electricity markets such as the New England ISO [58] or PJM [76] in the US use this locational marginal pricing scheme.

A locational marginal pricing approach involves a representation of the physical properties of the electricity network in the market-clearing procedure of the ISO. Electricity is a non-storable good and has very special physical properties. If electricity is generated at one node and extracted at another, the power flow will be dispersed over all paths between the two nodes. This electricity flow can be determined using Kirchhoff's voltage and current laws[8]. The implication of these physical laws is that the flow of power throughout a network is highly complex and does not obey any contractual path assumed by economic entities. Therefore, a detailed model of the network implies a model which is considerably complex.

1.1.4 On Equilibrium in a Pool-Based Electricity Market

In the new restructured power markets, different equilibrium models have been proposed by various authors to analyze the market. A review of equilibrium modeling approaches is presented in [12, 26, 50]. We identify a market equilibrium as the outcome of a market economy in which each market agent in the economy is doing as well as it can, given the actions of all the other market agents. A pool-based electricity market equilibrium can be defined as the levels of power transactions and the prices that satisfy simultaneously the next three properties:

1. Each generating company maximizes profits, taking prices as given.
2. Each consumer maximizes its utility, taking prices as given.
3. The market is cleared by the ISO maximizing social welfare.

In such an equilibrium, no market agent will want to alter its decision unilaterally. The prices obtained as a result of this equilibrium are called equilibrium prices.

A desirable electricity market equilibrium should consider the economic and technical features of the market agents as well as the network constraints. Market equilibrium models involving network constraints, such as transmission capacity limits are presented in [18, 52, 80, 92].

Note that several of the equilibrium models found in the literature, e.g. [50, 77], are solved using a linear complementarity problem that results from taking the Karush-Kuhn-Tucker conditions for each market agent problem.

On the other hand, under certain constraints there are some cases where equilibrium prices do not exist. Then, prices such that most economic requirements imposed by the market agents are satisfied can be found. In this scheme, we define a pool-based electricity market near-equilibrium as a set of prices and power transactions that simultaneously optimize problems for each market agent and entails sufficiently small infeasibilities.

1.2 Motivation and Contributions

The emerging restructured power industry is no longer modeled using the traditional electricity operation models where a central operator collects all the market agent data and centrally clears the market seeking to minimize the total operating costs. In this new scheme, market agents are competitive entities and are not necessarily interested in revealing some data, e.g. operating costs. New methodologies are appearing to help in modeling electricity markets as can be found in many reports and books. Boucher and Smeers [12] discuss various electricity market equilibrium proposals of the literature and establish correspondences between these different models. We can also find a classification and a review of the current developments made in modeling the electricity markets that calculate price equilibria in [26, 50, 60].

In an electricity market competitive equilibrium, market agents optimize the production, consumption and sales of electricity, while the production and consumption is balanced. This kind of equilibrium does not guarantee generating units recovering their costs. Therefore, some generating units might produce energy at a financial loss. This fact limits the equal opportunity for all generating companies to maximize their profits and to determine whether or not to operate in the market.

Including minimum profit requirements of the generating units as additional constraints of an equilibrium model is of practical importance since similar conditions are used in actual markets, such as the electricity market of mainland Spain [73]. Note that minimum profit conditions are complex to model because they involve both primal and dual variables. Relevant references on duality include [6, 21, 63]. In the electric energy literature, the number of references that develop equilibrium models allowing generating units to declare minimum profit conditions is very limited, the most relevant references are [67, 69].

Motto et al. [69] present a single-period electricity market equilibrium model that considers minimum profit conditions for the generating units. This model exploits the decomposable structure of the problem using an iterative Lagrangian relaxation algorithm [6]. Note that for some case studies, the model generates a cyclical behavior in prices, that is, it cannot find equilibrium prices.

Motto and Galiana [67] present a single-period equilibrium procedure that eliminates the cyclical behavior on prices that occurs when Lagrangian decomposition is used, [69]. To do so, Motto and Galiana [67] propose a form of augmented pricing that supports all the feasible points. The problem of the augmented pricing approach is that it modifies prices and includes several uplifts, so it is not clear that these prices accurately represent equilibrium prices. An uplift can be defined as a charge imposed on all customers that covers costs not covered by prices [85].

In references [67, 69], we observe that there are some cases where it is impossible to find equilibrium prices because they do not exist, so we cannot find a competitive equilibrium. In [69], this is shown by the cyclical behavior of prices, and in [67] by the need to include uplifts in prices.

Therefore, we define a new concept of equilibrium that may be more appropriate in electricity markets involving minimum profit conditions for the generating units. In a pool-based electricity market, the new equilibrium concept is achieved when every market agent maximizes its profit, all technical constraints including power balances are satisfied, and every scheduled generating unit satisfies its minimum profit condition. Due to the fact that equilibrium prices do not exist for some cases, we cannot calculate them in such cases. For these cases, the equilibrium model reported in this thesis obtains prices that cause slight infeasibilities in the equilibrium problem but that have a defendable interpretation as equilibrium prices. We refer to such slightly infeasible equilibrium as near-equilibrium and the prices as near-equilibrium prices.

There is another important issue in modeling an electricity market equilibrium. In general, a market equilibrium is achieved when each market agent simultaneously maximizes its profit. Therefore, we need a tool to simultaneously consider the conflicting points of view of the different agents. For this reason, we can use complementarity theory to find an electricity market equilibrium, as demonstrated in several references. We highlight previous works by Boucher and Smeers [12] and Hobbs [50]. The model presented in this thesis uses optimality conditions, that is, complementarity theory, to simultaneously solve the problem faced by every generating company, by every consumer and by the ISO, and this can be seen as an extension of [12, 50] in four respects. First, our model is more general in that it includes step functions but can also include a continuous demand function considered in [12] and [50]. Second, our model includes a detailed representation of the transmission network including the effect of both congestion and losses; losses were not considered in [12, 50]. Third, the model developed in this thesis satisfies minimum profit conditions for any generating unit that declares such a requirement and is not expelled from the market. By contrast, these conditions are not included in either [12] or [50]. Lastly, our model is extended to obtain a multi-period equilibrium with time-coupling constraints and with integer variables, that is, with indivisibilities (non-convexities). A single-period equilibrium is analyzed in [12, 50].

The modeling of non-convexities have largely been avoided due to the intractability of such problems, but as we can see in [87], assuming that non-convexities are unnecessary to model a market economy is unrealistic. For problems with integer constraints, one approach adopted in [74] is to solve the related optimization problem to optimality, then add constraints forcing the integer variables to be at the optimal levels. The result is a linear (or convex) program which has a defendable interpretation for an equilibrium. In the model we present, we make use of Benders decomposition to deal with indivisibilities. This decomposition procedure allows the calculation of integer variable values in a different problem from the main equilibrium problem, and these integer values are iteratively improved. Finally, the solution is optimal as regards both binary and continuous variables.

The equilibrium of the market is obtained using both complementarity theory and Benders decomposition. The combination of these two methodologies is relatively unstudied, but in this thesis we have shown that it works well.

Generally, the pricing mechanism applied in electricity markets is based on locational marginal pricing. Locational marginal prices are obtained as the dual variables associated with the balance constraints while maximizing social welfare in the problem of the ISO. It is important to be able to handle prices in the primal problem for including, for example, minimum profit conditions for the generating units or maximum cost conditions for the consumers. As we cannot directly impose constraints involving dual variables in the primal problem, we resort to complementarity theory to be able to do so. This is another important capability of the model we present.

Note that a procedure to identify the electricity market equilibrium is of interest for market regulators that may use it for market monitoring; and it is also of interest for the generating companies and the consumers to analyze their most appropriate strategies.

In summary, the main points that motivate the present work are:

1. To develop a tool that simultaneously takes into account the independent and conflicting viewpoints of all market agents.

2. To find equilibrium prices or near-equilibrium prices (in the case that equilibrium prices do not exist) that clear the market, simultaneously satisfying the objectives of each market agent, and the minimum profit conditions imposed by generating units.

3. To be able to include constraints in the equilibrium model that involve prices, i.e., dual variables.

4. To achieve points 1-3 within a multi-period framework that requires on / off decisions.

1.3 Problem Description

First, we formulate the problems faced by each market agent in the pool. We assume that the goal of each generating company is to maximize its profit subject to technical bounds on production, which involve on / off decisions and ramping limit constraints in a multi-period framework. The consumers can be modeled as maximizing their economic utilities while considering bounds on demand. Finally, the ISO carries out the market-clearing taking into account network constraints in order to achieve maximum social welfare. We consider that all the data are deterministic, not probabilistic, because we are working within a 24-hour time horizon, or, in other words, we analyze a short-term decision problem that does not include significant sources of uncertainty.

The generating companies and the consumers are modeled as solving appropriate linear optimization programs with locational marginal prices as inputs. The locational marginal prices are then determined as dual prices to balance constraints in the problem faced by the ISO.

We use realistic simplifications concerning line losses and power flows in the problem of the ISO, which we explain below. The transmission network can be represented in detail using an AC power flow model that includes non-linear equations. However, to achieve numerical tractability, we represent the electricity transmission network using a DC power flow that is a linear approximation. Using a DC power flow to represent the transmission network implies not taking electric energy losses into account. Nevertheless, we include a linearized version of losses in the power flow model [27, 30, 68, 93] to properly model losses.

In this work, we impose thermal limits to every line in the problem of the ISO. However, note that voltage and stability limits, either deterministic [66, 68] or probabilistic [13] can be incorporated into the model of the ISO. At the cost of increasing the computational burden, line and generating unit contingencies can also be incorporated into the model of the ISO as stated in [81].

The electricity market equilibrium is computed once the bidding stacks of every unit of each generating company and the bidding stacks of every demand of each consumer are submitted [27]. Note that price bids of the generating companies and the consumers are used by the ISO to maximize social welfare. However, these price bids may not coincide with the actual cost values and marginal utility values used by the generating companies and the consumers, respectively, in order to maximize their respective surplus.

The pool can be seen as a strategic game in which participants (generating companies and consumers) play against each other in order to maximize their own profits. The strategy that the participants follow is based on price bids. The behavior of the market participants can be simulated and analyzed using game theory [34, 37, 62]. The decision making process of the market participants in defining price bids is not the target of this thesis, therefore we assume that the bidding stacks of the generating companies and the consumers are data for the equilibrium problem.

The multi-period electricity market equilibrium being modeled is defined as the generating company / consumer energy transaction levels and their associated prices that result in maximum profit for every generating company, maximum utility for every consumer and maximum social welfare for the whole multi-period framework, while inter-temporal constraints including on / off status of the units and ramping limit constraints are enforced. Additionally, fixed, start-up and shut-down costs are considered [3, 89, 91].

In this scheme, the multi-period market equilibrium is obtained considering the set of linear and continuous optimization problems corresponding to the maximum profit / utility of the generating companies / consumers and the maximum social welfare of the ISO for a given time horizon.

Note that the multi-period electricity market equilibrium problem embodies binary decisions, i.e., on / off status for the units, and therefore optimality conditions cannot be directly applied to formulate this. The Benders decomposition technique is used to formulate the multi-period equilibrium problem avoiding binary variable limitations while retaining the advantages of using optimality conditions.

In this market equilibrium procedure, we can include conditions to impose minimum profit conditions by the generating units throughout the market horizon under study, eventually resulting in a market near-equilibrium. The units imposing such requirements could be expelled from the market if they render them uncompetitive.

For descriptive purposes and for clarity, we also consider the single-period market equilibrium procedure, which is still quite rich, in order to analyze the market equilibrium procedure developed in this thesis.

1.4 Solution Technique

Given the form of the individual market agent optimization problems, the Karush-Kuhn Tucker conditions [6] for the problems of the generating companies, the consumers and the ISO are both necessary and sufficient to obtain their respective solutions. The simultaneous solution of all these conditions constitutes a mixed linear complementarity problem [25, 32, 59] to be solved in order to determine the market equilibrium.

The multi-period market equilibrium problem includes continuous and binary variables and is solved using the Benders decomposition technique [7, 21, 43]. This technique decomposes the original problem into a master problem to compute the binary variable values and into a subproblem to obtain the market equilibrium corresponding to those binary values.

If binary variables are fixed to given values, the multi-period market equilibrium, corresponding to the status for the generating units defined by binary variables, can be solved as a quadratic programming problem. This problem is the Benders subproblem and is equivalent to a mixed linear complementarity problem [25, 59] derived from the optimality conditions of the problems for all the market agents. In turn, the master problem defines the on / off status for the generating units by obtaining the corresponding binary variables.

The solution of the subproblem provides useful information on the quality of the values of the binary variables related to the on / off status of the units, defined in the master problem. In turn, this information is used by the master problem to refine the on / off status for the generating units of the generating companies. This iterative procedure continues until some cost tolerance is reached providing the market equilibrium.

Note that we combine Benders decomposition and complementarity theory to achieve the solution of the multi-period electricity market equilibrium.

Minimum profit requirements for the generating units can be taken into account in order to solve the multi-period equilibrium / near-equilibrium problem. These conditions are included as additional constraints to the quadratic programming subproblem. Minimum profit conditions can be represented as bilinear equations and therefore convert the quadratic problem into a nonlinear one with nonlinear constraints, which is computationally difficult to solve. We propose three methods to solve this nonlinear problem. The first one is to directly solve the nonlinear subproblem using nonlinear solvers. The second one linearizes the bilinear minimum profit conditions resulting in just linear equations and binary variables. The third one obtains the solutions to the subproblem fixing selected variables in the bilinear equations, converting them into linear ones; then, we iteratively solve the quadratic subproblem updating the values of the fixed variables in the minimum profit conditions, until the solution satisfies equilibrium conditions and bilinear minimum profit conditions.

The market equilibrium can be obtained more easily for a single period of time. In this case, the market agent optimization problems do not involve binary constraints, so the Karush-Kuhn-Tucker conditions can be directly applied to obtain the equilibrium. To attain the market equilibrium considering minimum profit conditions for the generating units, the solution of the corresponding mixed linear complementarity problem is also obtained as an equivalent quadratic programming problem [25, 59], including minimum profit conditions as constraints to this quadratic problem. Finally, this quadratic problem is solved using either one of the three methods stated for the multi-period case.

1.5 Thesis Objectives

The main objectives of this thesis are stated below.

1. To develop a multi-period electricity market equilibrium procedure that obtains equilibrium prices, or near-equilibrium prices in the case that equilibrium prices do not exist.

2. To develop an equilibrium procedure that coordinates the independent and conflicting points of view of market agents to attain a solution in which no market agent wants to alter its decision unilaterally.

3. To formulate the multi-period electricity market equilibrium in such a way that allows including restrictions involving dual variables, i.e.,

prices, particularly minimum profit conditions imposed by generating units.

4. To propose methods to solve the multi-period electricity market equilibrium problem including bilinear constraints such as minimum profit conditions of the generating units.

5. To develop a solution procedure to handle the non-convexities associated with the multi-period equilibrium problem, such as costs and constraints related to on / off decisions.

6. To study the effect of imposing constraints involving prices for the generating units in the multi-period equilibrium model.

7. To study in particular, the single-period electricity market equilibrium so as in order to more clearly analyze the market equilibrium procedure developed in this thesis.

1.6 Literature Review

This section presents a review of the literature relevant to this thesis. References are grouped by subject.

1.6.1 References on Equilibrium

References [12, 26, 50, 60] present a review of electricity market equilibrium approaches, and [12] also establishes relationships between some of the equilibrium approaches, among others, [18, 52, 80, 92].

In this thesis, we develop an equilibrium model that includes minimum profit requirements for the generating units and provides equilibrium prices. In the case that equilibrium prices do not exist, our model finds near-equilibrium prices which entail slight infeasibilities for competitive markets with a reasonable behavior of the market agents. Including these requirements as additional constraints to an equilibrium model is of practical importance since similar conditions are used in actual markets, such as the electricity market of mainland Spain, [73]. In this context, we highlight [67, 69]. Motto and Galiana [67] discuss issues and methods for attaining the equilibrium in electric power auction markets with unit commitment and considering minimum profit conditions for the units. This paper shows that augmented pricing can coordinate self-interested agents, but it is not clear that these prices represent equilibrium prices. Motto et al. [69] present a single-period decentralized electricity market-clearing model that includes reactive power and demand responsiveness in addition to the more common framework of generation-side competition for electricity. This model considers minimum profit conditions for the units but presents a cyclical behavior on prices for some cases.

Our work can be seen as an extension to the previous work [12, 50]. Our model can also incorporate step demand functions, considers both congestion and losses, includes minimum profit conditions for the units, and models an equilibrium in a multi-period framework.

In this thesis, non-convexities associated with the time-coupling constraints in the multi-period equilibrium are dealt with using Benders decomposition, achieving an optimal solution in both binary and continuous variables. The existence of market-clearing prices in an economic analysis of a market with non-convexities is addressed in [74]. This paper solves the related optimization problem to optimality, then adds constraints forcing the integer variables to be at the optimal levels. The result is a linear program which has a defendable interpretation for an equilibrium.

If minimum profit requirements for the units are not included, in a centralized environment the model we present is equivalent to a multi-period optimal power flow. Relevant references of optimal power flow are [1, 5, 15, 54, 64, 91]. A multi-period optimal power flow, modeling inter-temporal constraints, is addressed in [1]. A generalized version of a unit commitment problem that includes thermal and hydro units is formulated in [5]. Carpentier [15] describes the development of the optimal power flow from its beginning. A survey of publications in the field of optimal power flow is presented in [54]. Ma and Shahidehpour [64] present how an optimal power flow with transmission security and voltage constraints is incorporated into the unit commitment formulation. Finally, Wood and Wollenberg [91] include a brief coverage of the security-constrained optimal power flow and its use in security control.

1.6.2 References on the Formulation of the Electricity Market Equilibrium Procedure

This thesis develops a model that simultaneously solves every market agent´s problem. Relevant references on modeling market agent behavior are considered below.

A detailed model of the generating units is developed in [3, 4, 16, 20, 89, 91]. An approach that allows a rigorous modeling of generators is proposed in [3]. A detailed formulation to model power trajectories followed by a thermal unit during the ramping limitations when increasing or decreasing power is presented in [4]. Reference [16] models hydro and thermal units in detail. The self-scheduling of a hydro generating company in a pool-based electricity market is addressed in [20]. A mathematical method for dealing with ramp-rate limits in unit commitment is proposed in [89]. Wood and Wollenberg [91] introduce and explore a number of engineering and economic matters involved in planning, operating and controlling power generation and transmission systems.

Relevant references to model the economic targets of market agents are [65, 86, 87, 88]. Mas-Colell et al. [65] explain the behavior of individual agents. A spatial price equilibrium model where a producer's optimal behavior only depends on current market prices is developed in [86]. Varian [87] provides a thorough treatment of optimization and equilibrium methods, detailing the behavior of the economic agents. Vives [88] provides a specialized characterization of the equilibrium. References [48, 49] describe methods to find the optimal bidding strategy of the market agents. These methods are usually based on game theory, [34, 37, 62]. Finally, market power in competitive markets is analyzed in references [11, 51].

In order to obtain an accurate market equilibrium we have modeled the network in detail. Relevant references on the network modeling are [8, 44, 46]. References [8, 44, 46] explore the major changes in the structure and operation of the electric utility industry due to electricity markets, and show how power system operation will be affected by the changes. Our representation of the transmission network includes the effect of congestion and a linearized version of losses similar to the one pioneered in [93] and used in [27, 30, 68]. References [33, 83, 84] define methods and tools for congestion management. An appropriate reference in order to incorporate voltage and stability limits is [66]. Bouffard and Galiana [13] provide a probabilistic security criterion that can be incorporated into the work reported in this thesis.

Note that the pricing mechanism used in our work is based on locational marginal prices because they are used in actual electricity markets such as [58, 76]. The pioneering work of Schweppe et al. [80] develop the basic theory and practical implementation issues associated with a spot price-based energy marketplace.

1.6.3 References on Techniques Used to Solve the Electricity Market Equilibrium Problem

The electricity market equilibrium problem can be formulated as a linear complementarity problem that simultaneously considers the point of view of every different agent. Bazaraa [6] deals with convex analysis, optimality conditions and duality, and provides appropriate background. References [50, 77] solve equilibrium models using a mixed complementarity problem that results from the set of Karush-Kuhn-Tucker conditions of each market agent problem. Information about duality can be found in [63].

Basic references on complementarity problems are [24, 25, 28, 32, 35, 59, 71]. A detailed study of the linear complementarity problem is provided in [25, 71]. A rigorous treatment of variational inequalities and complementarity problems in finite dimensions is presented in [32]. Several computational methods for solving complementarity problems are presented in [24, 28, 35, 59].

The multi-period equilibrium model formulated in the thesis includes non-convexities, so Benders decomposition algorithm is used to formulate and solve it. The Benders decomposition technique is explained in [7, 21, 43].

The addition of bilinear minimum profit constraints in the equilibrium problem turns this problem into a nonlinear program. Relevant references concerning the methods considered in this thesis in order to solve this kind of nonlinear problem are the following. A description of the Schur's decomposition is presented in [38, 53]. Winston [90] focuses on model-formulation and model-building. Iterative solutions of nonlinear systems of equations are covered in [23, 45, 72, 79].

1.6.4 References on the Case Studies

As part of this thesis, we apply the developed procedures to several power systems. The 4-node system used in the illustrative examples of Chapters 3 and 4 has been obtained from [46]. In Chapter 5, we analyze in detail the IEEE 24-node Reliability Test System, which is described in [47]. Details of this system can also be found in [9].

The solution to the optimization problems proposed throughout the thesis are obtained by using commercial solvers.

Linear complementarity problems can be solved using, among others, PATH [29], MILES [78] or SMOOTH [19]. A comparison of these solvers is given in [10]. ILOG [57] presents CPLEX, a well-known solver used to solve mixed-integer linear and quadratic programming problems. Reference [70] provides information on MINOS, a solver for nonlinear programming problems and [31] explains CONOPT, which also solves nonlinear problems.

It is convenient that these solvers work as part of a modeling language, as GAMS [39], AMPL [2] or AIMMS [75]. References [14] and [40] are a manual for the GAMS modeling language and for the solvers used by GAMS, respectively. The AMPL language is explained in [36].

1.6.5 References on Electricity Markets

This thesis has been developed considering an electricity market. Relevant information on electricity markets can be found in [17, 55, 56, 61, 81, 82, 85]. Chao and Peck [17] contain several papers that focus on how to design competitive electricity markets in an industry undergoing both rapid economic and technological changes. Reference [55] presents a review of the restructuring of the electric power industry. Ilic et al. [56] provide an overall perspective of changes that result from the restructuring of the electric power industry. Kirschen and Strbac [61] use a combination of traditional engineering techniques and fundamental economics to address the long-term problems of power system development in a competitive environment. Shahidehpour et al. [81] analyze the necessity and components of restructuring, explain the strategies of market participants, and propose techniques on how to recognize and evaluate the market risks. Sheblé [82] presents a complete description of the new industry structure as well as the various markets being formed. Stoft [85] presents the power-market design principles from economic theory to market architecture.

1.6.6 Publications Resulting from this Thesis

The material presented in this thesis has been published in [21, 22, 38, 41, 42].

Book [21] addresses decomposition in linear programming, mixed-integer linear programming, nonlinear programming, and mixed-integer nonlinear programming. It provides rigorous decomposition algorithms as well as heuristic ones. The book also provides practical applications in engineering and science.

Conejo et al. [22] compare two contrasting yet often used electricity market-clearing procedures: an auction-based algorithm including congestion management and transmission-loss cost allocation, and an optimal power flow method. These algorithms are compared in terms of the economic efficiency of the solution attained, and in terms of cross-subsidies between generators and demands. The purpose of this comparison is to quantify the actual cost to market participants of using a simple, seemingly transparent procedure, such as an auction-based algorithm, versus an integrated but computationally intensive one, such as an optimal power flow approach.

Gabriel et al. [38] provide a new methodology to solve bilinear, nonconvex mathematical programming problems by a suitable transformation of variables. Schur's decomposition and Special Ordered Sets (SOS) of type 2 variables are used resulting in a mixed-integer linear or quadratic program.

García-Bertrand et al. [41] provide a procedure to determine the near-equilibrium of an electricity market in a single-period. Conditions that ensure minimum profit for the generating units can be included. However, these conditions may render a generating unit uncompetitive and expel it from the market. The near-equilibrium is obtained through the solution of a mixed-integer quadratic problem equivalent to a mixed linear complementarity problem that includes the minimum profit conditions.

Finally, the concept of multi-period equilibrium is analyzed and illustrated in [42]. Within this equilibrium framework and a multi-period horizon, market agents simultaneously optimize their respective individual and conflicting objectives. Constraints involving prices can be incorporated into the problems of the market agents. To avoid the limitations imposed by the necessary use of binary variables to model on / off decisions, the conditions to attain a multi-period equilibrium are formulated through Benders decomposition, which allows for efficiently solving the resulting equilibrium problem.

1.7 Overview of Chapters

This document is organized as follows.

This introductory chapter provides an overview of electricity markets, emphasizing the roles of the market agents, how an electricity market based on a pool operates, and defines the concept of a market equilibrium / near-equilibrium. Next, the reasons that motivate this thesis are stated. We continue by describing the problems tackled in this thesis and propose several methods to solve them. Next, the main objectives of the thesis are listed, followed by a review of the literature. Finally, the organization of the thesis is presented.

In Chapter 2, we model the behavior of every market agent, namely, generating companies, consumers and the independent system operator. All the conditions pertaining to each agent are explained in detail.

Chapter 3 formulates the single-period electricity market equilibrium model and proposes a method to solve it. Then, the effect of imposing minimum profit conditions on the market equilibrium is discussed. Finally, we formulate the electricity market equilibrium / near-equilibrium model including such conditions and provide three solution techniques.

In Chapter 4, we extend the single-period equilibrium problem explained in Chapter 3 to a multi-period framework. This multi-period market equilibrium problem is formulated using Benders decomposition. Then, we provide a solution technique to solve this problem. As in Chapter 3, we also formulate the equilibrium model when minimum profit conditions are added to the problem, and propose an algorithm based on Benders decomposition to achieve the solution to the multi-period equilibrium problem.

Chapter 5 illustrates the proposed models and solutions techniques presented in Chapters 3 and 4 using several case studies. These case studies are analyzed and relevant results are reported.

In Chapter 6, we conclude this thesis providing several noteworthy conclusions and the main contributions of the work. Some suggestions regarding further research are also proposed.

Finally, this document includes four appendices. Appendix A provides a description of the linear complementarity problem. In Appendix B, we explain Benders decomposition. Data of the IEEE 24-node Reliability Test System used in Chapter 5 are provided in Appendix C. Appendix D collects additional results of the case studies analyzed in Chapter 5.

Chapter 2

Modeling of Market Agent Behavior

2.1 Introduction

This chapter presents models for each of the market agents. Generally, a pool-based electricity market includes generating companies, consumers and an Independent System Operator (ISO) or Market Operator (MO) [56, 81, 82]. Each generating company submits bids to the pool consisting of a set of energy production blocks and their corresponding minimum selling prices for every hour of the market horizon, and each consumer bids a set of consumption energy blocks and their corresponding maximum buying prices for every hour of the market horizon. In turn, the independent system operator clears the market by seeking maximum social welfare for the whole multi-period framework.

A generating company is the owner of one or more generating units located throughout the network, while a consumer exhibits one or more demands. More than one generating unit / demand can be located at each node of the network.

The generating companies and the consumers are modeled as solving appropriate optimization problems with Locational Marginal Prices (LMP) as inputs. These locational marginal prices are determined as the dual prices to balance constraints in the ISO optimization. The pricing mechanism used is based on locational marginal prices because they are used in actual electricity markets such as [58, 76].

In this framework, we consider that all the data on the generating companies, the consumers and the network are deterministic because the problem developed in this thesis considers a short-term horizon.

2.2 Generating Companies

For the sake of simplicity, it is supposed that the generating companies include only thermal generating units. The hydroelectric generating units can be modeled analogously as thermal units are modeled [20, 91].

A thermal generating unit has two states; on or off. These different states are modeled using binary variables which are referred to as on-line status variables. If the generating unit is on-line during the considered hour, the value of this variable is equal to 1; and if the generating unit is off-line, the value is 0.

Note that the start-up and shut-down status can be obtained from the on-line status in the time horizon.

The production of each generating unit is described using several power blocks with associated linear operating costs. This information is generally confidential. Thus, the production bids submitted to the pool may or may not correspond to the marginal costs depending on the bidding strategy of the generating company. The bidding strategy of any single generating company might be based on exploiting its potential market power, resulting in withholding power or raising bidding prices in order to raise equilibrium prices above competitive levels [27]. In recent years, there has been a great deal of discussion about how to best analyze the potential for market power in restructured markets [11, 51, 60], but this type of analysis is outside the scope of this thesis.

The generating companies are modeled assuming that the objective for each company is to maximize its total own profit subject to a set of operational constraints for the units which include those that link decisions in a period with decisions in the following and the previous periods [3, 4, 89, 91].

The next section explains the formulation of the objective function and the operational constraints of the set of the generating units of a generating company considering a multi-period framework.

2.2.1 Maximum Profit Criterion

In practice, a generating company seeks to maximize profits across all available markets [65]. However, this thesis concentrates on just the day-ahead market, which is the most relevant in terms of volume of trade. Profitability depends on the characteristics and costs of the generating unit in question, including start-up and shut-down costs, minimum up / down times, ramp rate limits and operating and maintenance costs. Profits also depend on the bidding strategy used by the generating company.

In this work, the generating units are assumed to incur operating, start-up and shut-down costs. It should be noted that we do not model the optimal bidding strategy of the generating companies; we assume that the bidding strategies of every company are known.

2.2.2 Objective Function

Each generating company seeks to maximize its total profit from producing and selling power in the market for the given time horizon. This profit includes revenues from selling energy in the market, as well as operating, start-up and shut-down costs from producing this energy.

2.2.2.1 Revenues

The type of market under consideration uses the concept of locational marginal pricing [80]. In this scheme, a generating unit injecting power at a given node is paid the locational marginal price corresponding to that node.

Consider a generating company f that owns the units indexed by the set G_f. Revenues obtained by this company for producing power on the multiperiod time horizon are computed as the product of the power generated by each unit and the locational marginal price corresponding to the node where the power is injected, for all units and for all the time periods considered. That is,

$$\sum_{t\in T}\sum_{i\in G_f}\rho_{n(i)}(t)\sum_{b=1}^{N_{Gi}}P_{Gib}(t), \tag{2.1}$$

where $\sum_{b=1}^{N_{Gi}}P_{Gib}(t)$ represents the total power produced by unit i at period t; and $\rho_{n(i)}(t)$ is the locational marginal price paid to unit i at period t which corresponds to the LMP at period t of node n where the unit i is located.

2.2.2.2 Costs

The power production by a generating unit entails a number of costs including operating, start-up and shut-down costs. These costs are modeled below.

- **Operating cost**

  The power production of a generating unit results in costs that depend on the generated power. These costs are due to fuel consumption, and to operation and maintenance. A precise model of the operation costs could require the use of non-differentiable and non-convex functions [91]. In practical applications, several approximations are used to model operating costs.

  The operating cost can be divided into a fixed cost, that is a constant value and independent of the generated power, and a variable cost that, in general, increases as generated power increases. In this work, a piecewise linear approximation of the variable cost is used.

  The generated power during each time period is assumed to remain constant. Taking the above into account, variable costs are linearized by blocks. The higher the number of blocks, the better the approximation is. The sum of the size of all power blocks considered is equal to the capacity (maximum power output) of the unit. Once the number and size of the blocks are determined, the marginal cost associated with each block can be computed.

Figure 2.1 shows the proposed approximation for the variable operating cost. Note that the number of blocks represented in the figure is four; therefore, operating cost are formulated using the same number of power variables, that is, $P_{Gi1}(t)$, $P_{Gi2}(t)$, $P_{Gi3}(t)$ and $P_{Gi4}(t)$. Each power variable has a minimum value equal to zero and a maximum value equal to the size of the corresponding block. The slope of the linear approximation for each block corresponds to the marginal cost of this block and is represented by $\lambda^{\mathrm{C}}_{Gi1}(t)$, $\lambda^{\mathrm{C}}_{Gi2}(t)$, $\lambda^{\mathrm{C}}_{Gi3}(t)$ and $\lambda^{\mathrm{C}}_{Gi4}(t)$. Note that the marginal cost increases as the produced power increases, therefore the cost function is convex.

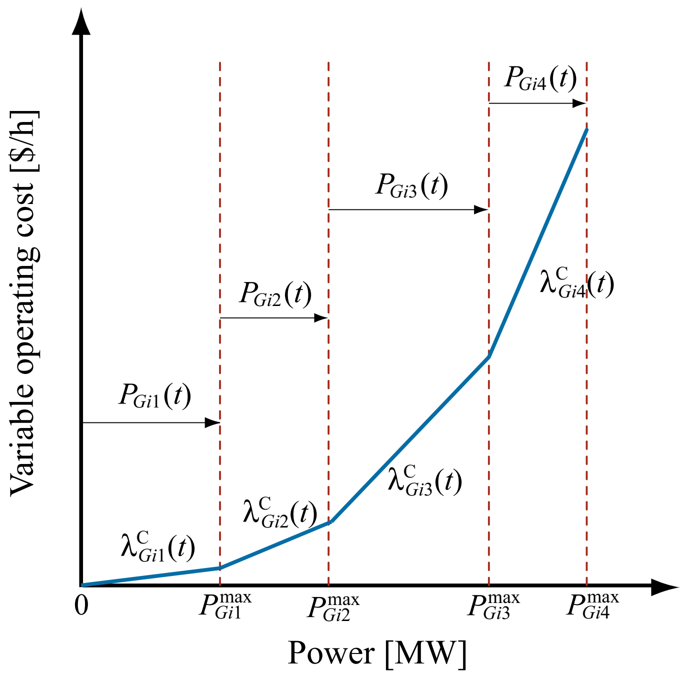

Figure 2.1: Piecewise linear variable operating cost function

The following expression represents total operating costs on the time horizon considered for a company f that owns the generating units indexed by the set G_f.

$$\sum_{t\in T}\sum_{i\in G_f}\left[C^{\mathrm{fx}}_{Gi}v_i(t)+\sum_{b=1}^{N_{Gi}}\lambda^{\mathrm{C}}_{Gib}(t)P_{Gib}(t)\right]. \tag{2.2}$$

Equation (2.2) represents the total operating cost of all units belonging to company f and for all considered time periods. The first term represents the fixed costs, C_{Gi}^{fx}, which are counted if the corresponding unit is on-line, that is if $v_i(t) = 1$; and the second term represents the variable costs that depend on the production level. The linear unit cost associated with each production block is represented by λ_{Gib}^{C}.

The following equations establish bounds to each power block of each unit at each time period.

$$P_{Gib}(t) \leq P_{Gib}^{\max}(t);\ \forall i \in G_f; b = 1, \ldots, N_{Gi}; \forall t \in T \tag{2.3}$$

$$P_{Gib}(t) \geq 0;\ \forall i \in G_f; b = 1, \ldots, N_{Gi}; \forall t \in T. \tag{2.4}$$

- **Start-up cost**

  If a unit is started up after being shut down for some time, the boiler must reach the correct working temperature and pressure. The costs incurred to obtain these conditions in the boiler are called start-up costs [91]. These costs are generally an exponential function of the hours that the unit has been shut down. The longer the time the boiler has been off, the more energy is needed to achieve the correct conditions in the boiler, and therefore the higher the start-up costs are. After a fixed number of hours off, the boiler temperature becomes constant as can be seen in Figure 2.2, therefore the start-up cost also becomes constant. For the sake of clarity and simplicity, this work considers that the start-up cost is constant and independent of the number of hours that the unit has been off-line.

  Consequently, the total start-up cost of a company f that owns several units during the considered time framework is provided by the expression,

$$\sum_{t \in T} \sum_{i \in G_f} c_{Gi}^{\text{su}}(t) \tag{2.5}$$

  enforcing that

$$c_{Gi}^{\text{su}}(t) \geq C_{Gi}^{\text{su}}\big[v_i(t) - v_i(t-1)\big];\ \forall i \in G_f; \forall t \in T \tag{2.6}$$

$$c_{Gi}^{\text{su}}(t) \geq 0;\ \forall i \in G_f; \forall t \in T, \tag{2.7}$$

  where $c_{Gi}^{\text{su}}(t)$ represents the start-up cost of the unit i at period t; and C_{Gi}^{su} is the constant start-up cost of unit i.

- **Shut-down cost**

  The costs incurred to shut down a generating unit are called shut-down costs. These costs are generally constant and is due to wasted fuel in the boiler during the shut-down process.

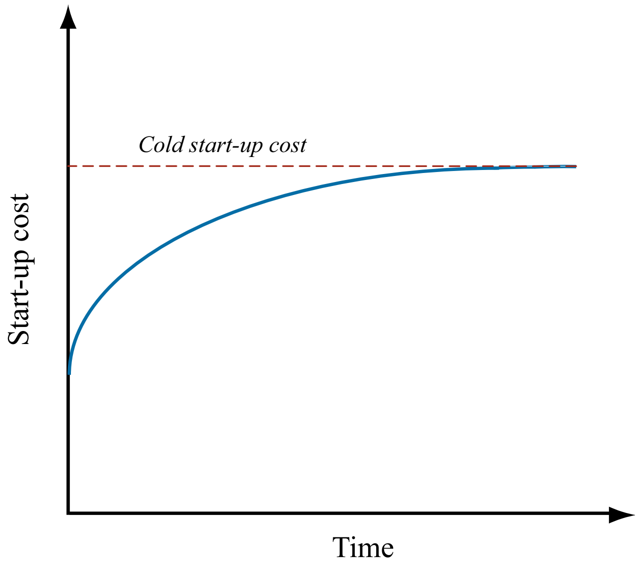

Figure 2.2: Evolution of start-up costs

The following expression represents the shut-down costs of a generating company f that owns several generating units for a multi-period framework,

$$\sum_{t \in T} \sum_{i \in G_f} c^{\mathrm{sd}}_{Gi}(t) \tag{2.8}$$

enforcing that

$$c^{\mathrm{sd}}_{Gi}(t) \geq C^{\mathrm{sd}}_{Gi}\big[v_i(t-1) - v_i(t)\big];\ \forall i \in G_f; \forall t \in T \tag{2.9}$$

$$c^{\mathrm{sd}}_{Gi}(t) \geq 0;\ \forall i \in G_f; \forall t \in T, \tag{2.10}$$

where $c^{\mathrm{sd}}_{Gi}(t)$ is the shut-down cost of the unit i at period t; and C^{sd}_{Gi} is the constant shut-down cost of unit i.

Taking into account the previous expressions, the total cost of a generating company f that owns several generating units indexed by the set G_f during the considered framework is

$$\sum_{t \in T} \sum_{i \in G_f} \Big[C^{\mathrm{fx}}_{Gi} v_i(t) + c^{\mathrm{su}}_{Gi}(t) + c^{\mathrm{sd}}_{Gi}(t) + \sum_{b=1}^{N_{Gi}} \lambda^{\mathrm{C}}_{Gib}(t) P_{Gib}(t)\Big] \tag{2.11}$$

enforcing that

$$P_{Gib}(t) \leq P_{Gib}^{\max}(t);\ \forall i \in G_f; b = 1, \ldots, N_{Gi}; \forall t \in T \tag{2.12}$$
$$P_{Gib}(t) \geq 0;\ \forall i \in G_f; b = 1, \ldots, N_{Gi}; \forall t \in T \tag{2.13}$$
$$c_{Gi}^{\text{su}}(t) \geq C_{Gi}^{\text{su}}\big[v_i(t) - v_i(t-1)\big];\ \forall i \in G_f; \forall t \in T \tag{2.14}$$
$$c_{Gi}^{\text{su}}(t) \geq 0;\ \forall i \in G_f; \forall t \in T \tag{2.15}$$
$$c_{Gi}^{\text{sd}}(t) \geq C_{Gi}^{\text{sd}}\big[v_i(t-1) - v_i(t)\big];\ \forall i \in G_f; \forall t \in T \tag{2.16}$$
$$c_{Gi}^{\text{sd}}(t) \geq 0;\ \forall i \in G_f; \forall t \in T. \tag{2.17}$$

2.2.3 Constraints

The operation of a generating unit is complex and the power output cannot take an arbitrary value in any given time period. There is a set of operational constraints that define the feasible region of the level of production. These constraints are explained in the following subsections and are imposed on each unit belonging to company f and for each period of the time horizon.

2.2.3.1 Capacity Limit

A generating unit has a capacity limit on production due to its design and the dimension of its components. This limit is established by the following expression

$$\sum_{b=1}^{N_{Gi}} P_{Gib}(t) \leq P_{Gi}^{\max} v_i(t);\ \forall i \in G_f; \forall t \in T, \tag{2.18}$$

where $P_{Gi}^{\max}$ represents the maximum power output of generating unit i.

2.2.3.2 Minimum Power Output

A generating unit is designed to work at or above a minimum power output in order to avoid combustion instability. This condition is imposed by the expression

$$\sum_{b=1}^{N_{Gi}} P_{Gib}(t) \geq P_{Gi}^{\min} v_i(t);\ \forall i \in G_f; \forall t \in T, \tag{2.19}$$

where $P_{Gi}^{\min}$ represents the minimum power output of the unit i.

2.2.3.3 Ramp Rate Limits

These constraints set the available maximum and minimum power output of a unit taking into account the start-up and shut-down ramp limits, and the ramp-up and ramp-down limits [16]. These constraints thus link any period with the following and the previous periods.

$$\sum_{b=1}^{N_{Gi}} P_{Gib}(t) - \sum_{b=1}^{N_{Gi}} P_{Gib}(t-1) \leq R_i^{\text{up}} v_i(t-1) + R_i^{\text{su}} \big[v_i(t) - v_i(t-1)\big]$$
$$+ P_{Gi}^{\max} \big[1 - v_i(t)\big]; \ \forall i \in G_f; \forall t \in T \tag{2.20}$$
$$\sum_{b=1}^{N_{Gi}} P_{Gib}(t-1) - \sum_{b=1}^{N_{Gi}} P_{Gib}(t) \leq R_i^{\text{dn}} v_i(t) + R_i^{\text{sd}} \big[v_i(t-1) - v_i(t)\big]$$
$$+ P_{Gi}^{\max} \big[1 - v_i(t-1)\big]; \ \forall i \in G_f; \forall t \in T. \tag{2.21}$$

Constraint (2.20) states that the available maximum power output of each generating unit during each hour depends on the ramp-up limit (R_i^{up}) in the case that the generating unit was running during the previous hour; or on the start-up ramp limit (R_i^{su}) in the case that the generating unit is started up at the beginning of the current hour. The last term of this constraint, $P_{Gi}^{\max}\big[1-v_i(t)\big]$, deactivates the limit on the available maximum power output in the case that the generating unit is shut down at the beginning of the current hour.

Constraint (2.21) states that the available minimum power output of each generating unit during each hour depends on the ramp-down limit (R_i^{dn}) in the case that the generating unit runs during the current hour; or on the shut-down ramp limit (R_i^{sd}) in the case that the generating unit is shut down at the beginning of the current hour. The last term of this constraint, $P_{Gi}^{\max}\big[1 - v_i(t-1)\big]$, deactivates the limit on the available minimum power output in the case that the generating unit is started up at the beginning of the current hour.

2.2.4 Formulation of the Problem of a Generating Company

Consider a generating company f that owns the units indexed by set G_f. This generating company chooses its production schedules by solving the following linear programming problem for the whole multi-period framework.

Maximize

$$\sum_{t\in T} \sum_{i\in G_f} \sum_{b=1}^{N_{Gi}} \Big[\big(\rho_{n(i)}(t) - \lambda_{Gib}^{\text{C}}(t)\big) P_{Gib}(t) - C_{Gi}^{\text{fx}} v_i(t) - c_{Gi}^{\text{su}}(t)$$
$$- c_{Gi}^{\text{sd}}(t) \Big] \tag{2.22}$$

subject to

$$\sum_{b=1}^{N_{Gi}} P_{Gib}(t) \leq P_{Gi}^{\max} v_i(t) : \ \alpha_i(t); \ \forall i \in G_f; \forall t \in T \tag{2.23}$$

$$\sum_{b=1}^{N_{Gi}} P_{Gib}(t) \geq P_{Gi}^{\min} v_i(t) : \ \beta_i(t); \ \forall i \in G_f; \forall t \in T \tag{2.24}$$

$$P_{Gib}(t) \leq P_{Gib}^{\max}(t) : \ \phi_{ib}(t); \ \forall i \in G_f; b = 1, \ldots, N_{Gi} - 1; \forall t \in T \tag{2.25}$$

$$\begin{aligned}\sum_{b=1}^{N_{Gi}} P_{Gib}(t) - \sum_{b=1}^{N_{Gi}} P_{Gib}(t-1) &\leq R_i^{\text{up}} v_i(t-1) + R_i^{\text{su}}\big[v_i(t) - v_i(t-1)\big] \\ &+ P_{Gi}^{\max}\big[1 - v_i(t)\big] : \ \tau_i(t); \ \forall i \in G_f; \forall t \in T\end{aligned} \tag{2.26}$$

$$\begin{aligned}\sum_{b=1}^{N_{Gi}} P_{Gib}(t-1) - \sum_{b=1}^{N_{Gi}} P_{Gib}(t) &\leq R_i^{\text{dn}} v_i(t) + R_i^{\text{sd}}\big[v_i(t-1) - v_i(t)\big] \\ &+ P_{Gi}^{\max}\big[1 - v_i(t-1)\big] : \ \psi_i(t); \ \forall i \in G_f; \forall t \in T\end{aligned} \tag{2.27}$$

$$P_{Gib}(t) \geq 0 : \ \eta_{ib}(t); \ \forall i \in G_f; b = 1, \ldots, N_{Gi}; \forall t \in T \tag{2.28}$$

$$c_{Gi}^{\text{su}}(t) \geq C_{Gi}^{\text{su}}\big[v_i(t) - v_i(t-1)\big]; \ \forall i \in G_f; \forall t \in T \tag{2.29}$$

$$c_{Gi}^{\text{su}}(t) \geq 0; \ \forall i \in G_f; \forall t \in T \tag{2.30}$$

$$c_{Gi}^{\text{sd}}(t) \geq C_{Gi}^{\text{sd}}\big[v_i(t-1) - v_i(t)\big]; \ \forall i \in G_f; \forall t \in T \tag{2.31}$$

$$c_{Gi}^{\text{sd}}(t) \geq 0; \ \forall i \in G_f; \forall t \in T. \tag{2.32}$$

The objective function (2.22) represents the total profit of the generating company f which is to be maximized subject to a capacity limit (2.23), a minimum power output (2.24) for each unit and each time period, a capacity limit (2.25) for each block of each unit and each time period except for the last block of each unit to avoid redundancy with constraint (2.23), the available maximum and minimum power output of a unit taking into account the start-up and shut-down ramp limits, and the ramp-up and ramp-down limits, (2.26) and (2.27) respectively, nonnegative levels of power to be generated by unit i in block b and time t, (2.28), and start-up and shut-down cost constraints for each generating unit at each time period, (2.29)-(2.32). Note that constraints (2.26) and (2.27) link a period with the following and the previous periods.

The dual variables of constraints (2.23), (2.24), (2.25), (2.26), (2.27) and (2.28) are respectively $\alpha_i(t)$, $\beta_i(t)$, $\phi_{ib}(t)$, $\tau_i(t)$, $\psi_i(t)$ and $\eta_{ib}(t)$. Note that the dual variables to constraints appear to the right of these constraints following colon in the problem formulations.

The above model is kept simple; however, it could easily incorporate additional features such as minimum up and down time constraints [89], contribution to the spinning reserve of the system [89], fuel choice options and emission allowances, [91]. It is presented in its current form for the sake of clarity.

Note that we can model the problem of a generating company that owns hydroelectric units by including additional constraints to this formulation, such as water balance constraints, reservoir level limits, allowed discharge limits and the energy conversion function [20, 91].

2.2.5 First Order Optimality Conditions

The problem formulated in the previous subsection, problem (2.22)-(2.32), includes both binary and continuous variables and therefore Karush-Kuhn-Tucker (KKT) optimality conditions cannot be directly applied to solve this. To avoid such limitations, the binary variables are fixed to given values. Chapter 4 explains how this assumption is made compatible with the solution of the original problem that includes binary variables.

If binary variables are fixed to given values, the solution of a generating company problem can be obtained by solving its first order optimality conditions [6]. These first order optimality conditions are KKT conditions expressed as a linear complementarity problem [25]. Note that the KKT optimality conditions [6] are both necessary and sufficient for describing optimal points because the problem of a generating company is a linear programming problem.

The optimality conditions for the problem of the generating company f, problem (2.22)-(2.32), decompose by unit due to the fact that no condition links different units of the same generating company. These conditions can be formulated as finding generation power blocks levels $P_{Gib}(t)$ and dual variables $\alpha_i(t)$, $\beta_i(t)$, $\phi_{ib}(t)$, $\tau_i(t)$, $\psi_i(t)$ and $\eta_{ib}(t)$ such that,

$$\begin{aligned}0 = {} & \lambda^{\mathrm{C}}_{Gib}(t) - \rho_{n(i)}(t) + \alpha_i(t) - \beta_i(t) + \phi_{ib}(t) + \tau_i(t) - \tau_i(t-1) \\ & + \psi_i(t-1) - \psi_i(t) - \eta_{ib}(t);\ \forall i \in G_f; b = 1, \ldots, N_{Gi}; \forall t \in T \end{aligned} \tag{2.33}$$

$$0 \le P^{\max}_{Gi} v_i(t) - \sum_{b=1}^{N_{Gi}} P_{Gib}(t) \perp \alpha_i(t) \ge 0;\ \forall i \in G_f; \forall t \in T \tag{2.34}$$

$$0 \le \sum_{b=1}^{N_{Gi}} P_{Gib}(t) - P^{\min}_{Gi} v_i(t) \ \perp \beta_i(t) \ge 0;\ \forall i \in G_f; \forall t \in T \tag{2.35}$$

$$\begin{aligned}0 \le {} & P^{\max}_{Gib}(t) - P_{Gib}(t) \ \perp\ \phi_{ib}(t) \ge 0;\ \forall i \in G_f; b = 1, \ldots, N_{Gi} - 1; \\ & \forall t \in T\end{aligned} \tag{2.36}$$

$$\begin{aligned}0 \le {} & R^{\mathrm{up}}_i v_i(t-1) + R^{\mathrm{su}}_i \big[v_i(t) - v_i(t-1)\big] + P^{\max}_{Gi}\big[1 - v_i(t)\big] \\ & - \sum_{b=1}^{N_{Gi}} P_{Gib}(t) + \sum_{b=1}^{N_{Gi}} P_{Gib}(t-1) \ \perp\ \tau_i(t) \ge 0;\ \forall i \in G_f; \forall t \in T\end{aligned} \tag{2.37}$$

$$\begin{aligned}0 \le {} & R^{\mathrm{dn}}_i v_i(t) + R^{\mathrm{sd}}_i \big[v_i(t-1) - v_i(t)\big] + P^{\max}_{Gi}\big[1 - v_i(t-1)\big] \\ & - \sum_{b=1}^{N_{Gi}} P_{Gib}(t-1) + \sum_{b=1}^{N_{Gi}} P_{Gib}(t) \ \perp\ \psi_i(t) \ge 0;\ \forall i \in G_f; \forall t \in T\end{aligned} \tag{2.38}$$

$$0 \leq P_{Gib}(t) \ \perp\ \eta_{ib}(t) \geq 0;\ \forall i \in G_f; b = 1, \ldots, N_{Gi}; \forall t \in T \tag{2.39}$$

$$0 \leq c^{\mathrm{su}}_{Gi}(t) - C^{\mathrm{su}}_{Gi}\big[v_i(t) - v_i(t-1)\big];\ \forall i \in G_f; \forall t \in T \tag{2.40}$$

$$0 \leq c^{\mathrm{su}}_{Gi}(t);\ \forall i \in G_f; \forall t \in T \tag{2.41}$$

$$0 \leq c^{\mathrm{sd}}_{Gi}(t) - C^{\mathrm{sd}}_{Gi}\big[v_i(t-1) - v_i(t)\big];\ \forall i \in G_f; \forall t \in T \tag{2.42}$$

$$0 \leq c^{\mathrm{sd}}_{Gi}(t);\ \forall i \in G_f; \forall t \in T. \tag{2.43}$$

By convention, the symbol $\perp$ indicates that one of the inequalities is satisfied as an equality, so the product of each equation and the corresponding variable must be zero, i.e., $0 \leq x \perp y \geq 0$ is equivalent to $x\, y = 0$, $0 \leq x$ and $0 \leq y$; this product is called the complementarity condition.

Equations (2.33)-(2.43) are the optimality conditions of the problem of the generating company f and are identical for all generating companies. This is so because we consider that the behavior of any generating company is similar to and independent of the rest of the companies. Therefore, the conditions below comprise all units of all generating companies. Note that these conditions are similar to equations (2.33)-(2.43), but including all the generating companies.

$$\begin{aligned} 0 = {} & \lambda^{\mathrm{C}}_{Gib}(t) - \rho_{n(i)}(t) + \alpha_i(t) - \beta_i(t) + \phi_{ib}(t) + \tau_i(t) - \tau_i(t-1) \\ & + \psi_i(t-1) - \psi_i(t) - \eta_{ib}(t);\ \forall i \in G; b = 1, \ldots, N_{Gi}; \forall t \in T \end{aligned} \tag{2.44}$$

$$0 \leq P^{\max}_{Gi} v_i(t) - \sum_{b=1}^{N_{Gi}} P_{Gib}(t) \ \perp \alpha_i(t) \geq 0;\ \forall i \in G; \forall t \in T \tag{2.45}$$

$$0 \leq \sum_{b=1}^{N_{Gi}} P_{Gib}(t) - P^{\min}_{Gi} v_i(t) \ \perp \beta_i(t) \geq 0;\ \forall i \in G; \forall t \in T \tag{2.46}$$

$$\begin{aligned} 0 \leq {} & P^{\max}_{Gib}(t) - P_{Gib}(t) \ \perp\ \phi_{ib}(t) \geq 0;\ \forall i \in G; b = 1, \ldots, N_{Gi} - 1; \\ & \forall t \in T \end{aligned} \tag{2.47}$$

$$\begin{aligned} 0 \leq {} & R^{\mathrm{up}}_i v_i(t-1) + R^{\mathrm{su}}_i \big[v_i(t) - v_i(t-1)\big] + P^{\max}_{Gi}\big[1 - v_i(t)\big] \\ & - \sum_{b=1}^{N_{Gi}} P_{Gib}(t) + \sum_{b=1}^{N_{Gi}} P_{Gib}(t-1) \ \perp\ \tau_i(t) \geq 0;\ \forall i \in G; \forall t \in T \end{aligned} \tag{2.48}$$

$$\begin{aligned} 0 \leq {} & R^{\mathrm{dn}}_i v_i(t) + R^{\mathrm{sd}}_i \big[v_i(t-1) - v_i(t)\big] + P^{\max}_{Gi}\big[1 - v_i(t-1)\big] \\ & - \sum_{b=1}^{N_{Gi}} P_{Gib}(t-1) + \sum_{b=1}^{N_{Gi}} P_{Gib}(t) \ \perp\ \psi_i(t) \geq 0;\ \forall i \in G; \forall t \in T \end{aligned} \tag{2.49}$$

$$0 \leq P_{Gib}(t) \ \perp\ \eta_{ib}(t) \geq 0;\ \forall i \in G; b = 1, \ldots, N_{Gi}; \forall t \in T \tag{2.50}$$

$$0 \leq c^{\mathrm{su}}_{Gi}(t) - C^{\mathrm{su}}_{Gi}\big[v_i(t) - v_i(t-1)\big];\ \forall i \in G; \forall t \in T \tag{2.51}$$

$$0 \leq c^{\mathrm{su}}_{Gi}(t);\ \forall i \in G; \forall t \in T \tag{2.52}$$

$$0 \leq c^{\mathrm{sd}}_{Gi}(t) - C^{\mathrm{sd}}_{Gi}\big[v_i(t-1) - v_i(t)\big];\ \forall i \in G; \forall t \in T \tag{2.53}$$

$$0 \leq c^{\mathrm{sd}}_{Gi}(t);\ \forall i \in G; \forall t \in T. \tag{2.54}$$

System of equations (2.44)-(2.54) can be reduced by eliminating the variable $\eta_{ib}(t)$. Then, equations (2.44) and (2.50) are reduced to equation (2.55) below. The resulting system is

$$\begin{aligned}
&0 \leq \lambda^{\mathrm{C}}_{Gib}(t) - \rho_{n(i)}(t) + \alpha_i(t) - \beta_i(t) + \phi_{ib}(t) + \tau_i(t) - \tau_i(t-1) \\
&\quad + \psi_i(t-1) - \psi_i(t) \ \perp \ P_{Gib}(t) \geq 0; \ \forall i \in G; b = 1, \ldots, N_{Gi}; \\
&\quad \forall t \in T && (2.55)
\end{aligned}$$

$$0 \leq P^{\max}_{Gi} v_i(t) - \sum_{b=1}^{N_{Gi}} P_{Gib}(t) \ \perp \ \alpha_i(t) \geq 0; \ \forall i \in G; \forall t \in T \qquad (2.56)$$

$$0 \leq \sum_{b=1}^{N_{Gi}} P_{Gib}(t) - P^{\min}_{Gi} v_i(t) \ \perp \ \beta_i(t) \geq 0; \ \forall i \in G; \forall t \in T \qquad (2.57)$$

$$\begin{aligned}
&0 \leq P^{\max}_{Gib}(t) - P_{Gib}(t) \ \perp \ \phi_{ib}(t) \geq 0; \ \forall i \in G; b = 1, \ldots, N_{Gi} - 1; \\
&\quad \forall t \in T && (2.58)
\end{aligned}$$

$$\begin{aligned}
&0 \leq R^{\mathrm{up}}_i v_i(t-1) + R^{\mathrm{su}}_i \big[v_i(t) - v_i(t-1)\big] + P^{\max}_{Gi} \big[1 - v_i(t)\big] \\
&\quad - \sum_{b=1}^{N_{Gi}} P_{Gib}(t) + \sum_{b=1}^{N_{Gi}} P_{Gib}(t-1) \ \perp \ \tau_i(t) \geq 0; \ \forall i \in G; \forall t \in T && (2.59)
\end{aligned}$$

$$\begin{aligned}
&0 \leq R^{\mathrm{dn}}_i v_i(t) + R^{\mathrm{sd}}_i \big[v_i(t-1) - v_i(t)\big] + P^{\max}_{Gi} \big[1 - v_i(t-1)\big] \\
&\quad - \sum_{b=1}^{N_{Gi}} P_{Gib}(t-1) + \sum_{b=1}^{N_{Gi}} P_{Gib}(t) \ \perp \ \psi_i(t) \geq 0; \ \forall i \in G; \forall t \in T && (2.60)
\end{aligned}$$

$$0 \leq c^{\mathrm{su}}_{Gi}(t) - C^{\mathrm{su}}_{Gi} \big[v_i(t) - v_i(t-1)\big]; \ \forall i \in G; \forall t \in T \qquad (2.61)$$

$$0 \leq c^{\mathrm{su}}_{Gi}(t); \ \forall i \in G; \forall t \in T \qquad (2.62)$$

$$0 \leq c^{\mathrm{sd}}_{Gi}(t) - C^{\mathrm{sd}}_{Gi} \big[v_i(t-1) - v_i(t)\big]; \ \forall i \in G; \forall t \in T \qquad (2.63)$$

$$0 \leq c^{\mathrm{sd}}_{Gi}(t); \ \forall i \in G; \forall t \in T. \qquad (2.64)$$

Note that similar simplifications in the KKT conditions (i.e. equations (2.44) and (2.50) replaced by equation (2.55)) are carried out as regards the problems of the consumers and the ISO, which are considered below.

2.3 Consumers

Each consumer has a set of demands and their consumption is described using several power blocks with associated linear utilities. Utility represents consumer satisfaction on using electricity. Based on this information, the consumer decides the bidding strategy for each demand, that is, the quantities and prices to submit to the market. These demand bids are submitted to the ISO and may not coincide with the corresponding utilities. Note that we refer to large consumers, that is, industrial or commercial consumers. Consumers might exercise their potential oligopsony power in deriving their bidding stacks [27], but we consider bidding stacks to be data in our problem; therefore,the way to compute demand bids is outside the scope of this thesis.

In this section, any consumer is modeled as maximizing its own profit considering that a minimum demand requirement must be satisfied. The formulation of the problem of a typical consumer, including the objective function and the associated constraints, is described below.

2.3.1 Maximum Utility Criterion

The consumer preferences are summarized by means of a utility function [65], therefore an appropriate mathematical program can be used to solve the problem of the consumer.

The consumer behavior is represented considering that the consumer chooses a consumption level to maximize its utility level minus its consumption costs.

2.3.2 Objective Function

A consumer seeks to maximize the total profit from consuming electricity in the market for a given time frame. This profit is obtained as the difference of the utility associated with the consumption of energy and the corresponding demand costs [65, 88].

2.3.2.1 Utility

The consumer is assumed to have a rational behavior and the utility function representing that behavior is nonlinear. Utility is linearized by blocks using marginal utilities. The marginal utility represents the satisfaction increase for consuming an additional MWh. This marginal utility decreases as the consumed power increases, therefore the utility function is concave [65, 87]. This fact is illustrated in Figure 2.3.

Consider a consumer q that has the demands indexed by the set D_q. The utility of this consumer for the consumption of the demands on the market horizon is represented by the following expression,

$$\sum_{t \in T} \sum_{j \in D_q} \sum_{k=1}^{N_{Dj}} \lambda^{\mathrm{U}}_{Djk}(t) P_{Djk}(t), \tag{2.65}$$

where $\lambda^{\mathrm{U}}_{Djk}(t)$ represents the marginal utility associated with block k of demand j at period t. The following equations (2.66) and (2.67) are the bounds of the consumed power in each block of each demand at each time period.

$$P_{Djk}(t) \le P^{\max}_{Djk}(t);\ \forall j \in D_q; k = 1, \dots, N_{Dj}; \forall t \in T \tag{2.66}$$

$$P_{Djk}(t) \ge 0;\ \forall j \in D_q; k = 1, \dots, N_{Dj}; \forall t \in T. \tag{2.67}$$

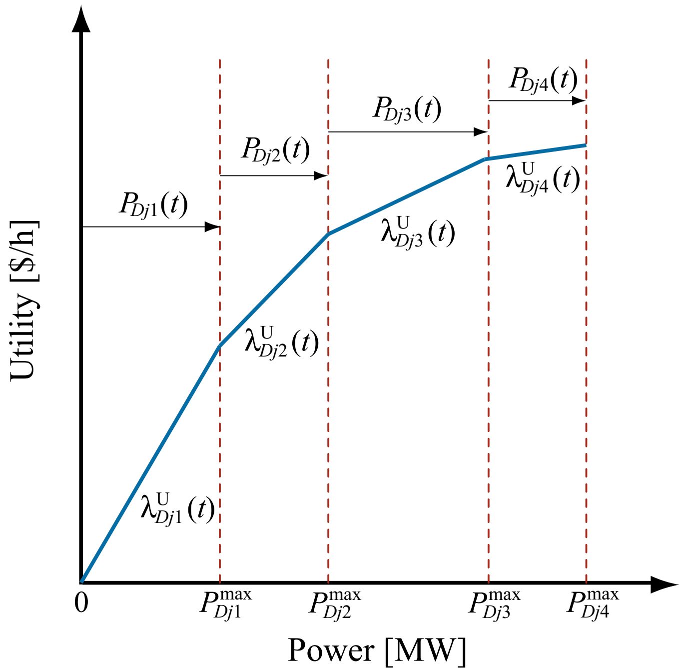

Figure 2.3: Piecewise linear utility function

2.3.2.2 Demand Costs

The market under consideration works under locational marginal pricing [80]. Then, a demand load receiving power from a given node pays the locational marginal price corresponding to that node.

Considering a consumer q whose set of demands is included in set D_q, costs for consuming power throughout the multi-period framework by this consumer are computed as the product of the power consumed by each demand and the corresponding locational marginal price. Demand costs are computed by the expression that is shown below,

$$\sum_{t\in T}\sum_{j\in D_q}\rho_{n(j)}(t)\sum_{k=1}^{N_{Dj}}P_{Djk}(t), \tag{2.68}$$

where $\sum_{k=1}^{N_{Dj}}P_{Djk}(t)$ represents the total power consumed by demand j at period t; and $\rho_{n(j)}(t)$ is the locational marginal price paid by demand j at period t which corresponds to the LMP at period t of the node where demand j is located.

2.3.3 Constraints

In this work, the only constraint that is enforced for each consumer is a minimum demand requirement. This is explained below.

2.3.3.1 Minimum Demand Requirement

As electricity is an essential good, each demand requires a minimum amount of consumption. This fact is represented through the following expression called minimum demand requirement.

$$\sum_{k=1}^{N_{Dj}} P_{Djk}(t) \geq P_{Dj}^{\min}(t);\ \forall j \in D_q; \forall t \in T, \tag{2.69}$$

where $P_{Dj}^{\min}(t)$ is the minimum power supplied to the demand j in period t.

We do not explicitly impose a maximum amount of consumption but note that through equation (2.66) we implicitly establish this limit.

2.3.4 Formulation of the Problem of the Consumers

We consider below the problem of the consumer q whose demands are included in set D_q. We assume that such a consumer can be modeled as maximizing its economic profit for the whole multi-period framework as stated in previous subsections, resulting in the following linear programming formulation.

Maximize

$$\sum_{t\in T}\sum_{j\in D_q}\sum_{k=1}^{N_{Dj}}\big(\lambda_{Djk}^{\mathrm{U}}(t) - \rho_{n(j)}(t)\big)P_{Djk}(t) \tag{2.70}$$

subject to

$$\sum_{k=1}^{N_{Dj}} P_{Djk}(t) \geq P_{Dj}^{\min}(t) :\ \sigma_j(t);\ \forall j \in D_q; \forall t \in T \tag{2.71}$$

$$P_{Djk}(t) \leq P_{Djk}^{\max}(t) :\ \varphi_{jk}(t);\ \forall j \in D_q; k = 1, \dots, N_{Dj}; \forall t \in T \tag{2.72}$$

$$P_{Djk}(t) \geq 0;\ \forall j \in D_q; k = 1, \dots, N_{Dj}; \forall t \in T. \tag{2.73}$$

The objective function (2.70) represents the economic utility for consumer q. Equation (2.71) represents the minimum load that must be supplied, with the dual variable of this equation being $\sigma_j(t)$. Equation (2.72) represents the maximum power in each block of each demand and each time period, and its dual variable is $\varphi_{jk}(t)$. Equation (2.73) imposes the constraint that the power to be consumed by demand j in block k in time t is nonnegative.

2.3.5 First Order Optimality Conditions

As the problem of a consumer is a linear programming problem, the KKT optimality conditions [6] are both necessary and sufficient for describing optimal points. Therefore, the optimal consumption of the demands of a consumer can be obtained by solving the linear complementarity problem corresponding to the KKT optimality conditions of the consumer problem, (2.70)-(2.73).

The optimality conditions for this optimization problem decompose by demand and by time period because no condition includes information about two or more demands and time periods. They can be formulated as finding demand power blocks $P_{Djk}(t)$, and dual variables $\sigma_j(t)$ and $\varphi_{jk}(t)$ such that,

$$
\begin{aligned}
&0 \le \rho_{n(j)}(t) - \lambda^{\mathrm{U}}_{Djk}(t) - \sigma_j(t) + \varphi_{jk}(t) \ \perp\ P_{Djk}(t) \ge 0;\ \forall j \in D_q;\\
&\quad k = 1, \ldots, N_{Dj}; \forall t \in T \qquad (2.74)\\
&0 \le \sum_{k=1}^{N_{Dj}} P_{Djk}(t) - P^{\min}_{Dj}(t) \ \perp\ \sigma_j(t) \ge 0;\ \forall j \in D_q; \forall t \in T \qquad (2.75)\\
&0 \le P^{\max}_{Djk}(t) - P_{Djk}(t) \ \perp\ \varphi_{jk}(t) \ge 0;\ \forall j \in D_q; k = 1, \ldots, N_{Dj};\\
&\quad \forall t \in T. \qquad (2.76)
\end{aligned}
$$

Note that the dual variable of equation (2.73) has been eliminated using the same simplification made for the generating companies, in Subsection 2.2.5.

Equations (2.74)-(2.76) are the optimality conditions of all demands of consumer q. The behavior of all consumers are considered to be similar and independent of each other, so the above optimality conditions can be generalized for all consumers. Therefore, the conditions below comprise all demands of all consumers.

$$
\begin{aligned}
&0 \le \rho_{n(j)}(t) - \lambda^{\mathrm{U}}_{Djk}(t) - \sigma_j(t) + \varphi_{jk}(t) \ \perp\ P_{Djk}(t) \ge 0;\ \forall j \in D;\\
&\quad k = 1, \ldots, N_{Dj}; \forall t \in T \qquad (2.77)\\
&0 \le \sum_{k=1}^{N_{Dj}} P_{Djk}(t) - P^{\min}_{Dj}(t) \ \perp\ \sigma_j(t) \ge 0;\ \forall j \in D; \forall t \in T \qquad (2.78)\\
&0 \le P^{\max}_{Djk}(t) - P_{Djk}(t) \ \perp\ \varphi_{jk}(t) \ge 0;\ \forall j \in D; k = 1, \ldots, N_{Dj};\\
&\quad \forall t \in T. \qquad (2.79)
\end{aligned}
$$

2.4 The Independent System Operator

A competitive electricity market includes profit-maximization entities such as generating companies, consumers and marketers. A coordinator, independent of the market participants, is required for the appropriate working of the market [17, 56, 81, 82]. Although the responsibilities of the system coordinator differ between models; in general, the ISO is set up to guarantee a non-discriminating access of market participants to the market.

Depending on the market, coordination can be developed by different entities that basically are a market operator or / and an independent system operator. The responsibilities of each one of these entities are the following:

- **Market operator:** With the purpose of ensuring the proper operation of the market, this entity assumes the functions required to perform the financial management and, in particular, the management of the electric power purchase and sale bids.

- **Independent system operator:** The entity that is responsible for the physical control of the system to maintain its security and reliability.

In some market structures, the MO and the ISO are separate entities. However, in other structures, the MO function is within the same organization and under the control of the ISO, therefore, the ISO is also responsible for the financial management.

The electricity market considered in this work is based on a pool. This market is modeled as having an independent system operator that clears the market and is responsible for the financial management of the market as well as the management of the transmission network. The targets of the ISO are to enforce transmission capacity limits, to maintain independence from the market participants, to avoid discrimination against the market participants, and to promote the efficiency of the market.

The electricity market modeled works as follows: First, each generating company sends the bidding stacks of each of its units to the pool and each consumer sends the bidding stacks of each of its demands to the pool. Then, the ISO clears the market using an appropriate market-clearing procedure resulting in prices, and production and consumption schedules.

The market-clearing procedure may embody network constraints, which model losses [27, 30, 68, 93] and line capacity limits [33, 83, 84], or not. In this work we model network constraints, and the resulting prices are therefore locational marginal prices [80]. In this scheme, a generating unit injecting power at a given node is paid the locational marginal price corresponding to that node; and conversely, a demand receiving power from a given node pays the locational marginal price corresponding to that node.

From a modeling point of view, there are some features associated with the transmission network that must be taken into account such as:

- **Losses:** Losses are incurred in the transmission (and distribution) network as the electricity flows to the consumers. Most of these losses are attributable to the heating of the power lines by the electrical current flowing through them.

- **Line capacity limits:** Due to technical reasons, power flow through any line is limited. If a line is working under its maximum flow, it is said that the line is congested. Line congestion can limit production of some generating units and / or consumption of the demands, producing technical and economic inefficiencies.

In this work, we consider a network representation of the electric power system consisting in a linearized power flow model that includes a precise representation of losses. This linear model makes it possible to compute locational marginal prices while taking into account the effects of line congestion and transmission losses in an accurate and efficient manner.

For the sake of clarity, we first formulate the market-clearing procedure used by the ISO to clear the market without modeling losses. Then, this procedure is extended to include a linearized version of losses.

2.4.1 Maximum Social Welfare

The market participants have different goals. The coordinator of the market, that is, the independent system operator, has responsibility for setting up transactions between market participants, as well as maximizing social welfare [86].

Social welfare is the total benefit realized by the transactions [65] and can be defined as the sum of the consumer surplus and the producer surplus, and is represented by the shaded area in Figure 2.4. The consumer surplus is computed as the difference of the sum of accepted demand bids times their corresponding bid prices and the costs for consuming these accepted demand bids. Similarly, the producer surplus is computed as the difference of the revenues for producing the accepted production bids and the sum of accepted production bids times their corresponding bid prices. It should be noted that if the generating companies do not bid at their respective marginal costs, the ISO does not strictly maximize the social welfare.

The ISO clears the market by seeking maximum social welfare for the entire time horizon. To do so, the ISO must know the bidding stacks of the generating units and the demands to clear the market. In what follows, the bidding rules of the generating units and the demands are described.

2.4.1.1 Bidding Rules

A bid is a specified amount of electricity at a given price. Most common bid-based pool markets require step-functions. This is the reason for using step-functions in the model presented. However, other bidding curves, such as continuous curves, can be straightforwardly incorporated into the model [48, 49].

Note that we are not interested in determining the bidding strategy of the generating companies and the consumers, because we assume that bids

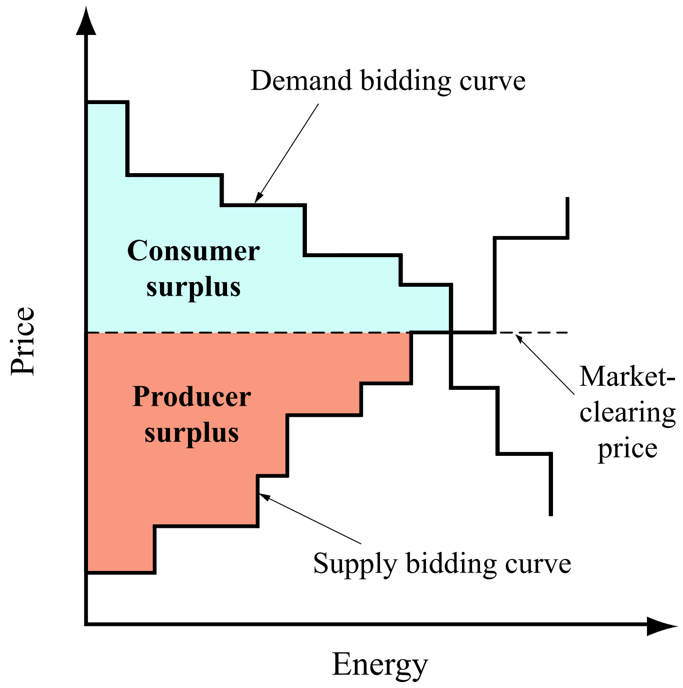

Figure 2.4: Social welfare

are data in our problem. Game theory [34, 37, 62] can be used to simulate the decision making process of the generating companies and the consumers in a pool-based electricity market.

The generating companies submit electric power sale bids to the pool for each of the generating units they own and for all hourly scheduling periods. For each hour, bids of each generating unit are monotonically increasing piecewise constant stacks of quantities and prices. Note that prices are not necessarily marginal costs [65, 80, 87].

Analogously, the consumers submit electric power purchase bids to the pool for each of the demands they own and for the hourly scheduling periods. For each hour, bids of each demand are monotonically decreasing piecewise constant stacks of quantities and prices. Note that prices do not necessarily represent marginal utilities [65, 80, 87].

For a given time period, Figure 2.4 shows supply and demand step-functions.

2.4.2 Objective Function

The ISO clears the market by seeking maximum social welfare. The social welfare can be expressed as the difference of two terms. The first term is the sum of the power blocks bid and accepted by each demand in each period, multiplied by the corresponding bidding price. The second term is the sum of the power blocks bid and accepted by each generating unit in each period, multiplied by the corresponding bidding price. The following expression represents the social welfare of a market for all considered time periods.

$$\sum_{t \in T} \sum_{j \in D} \sum_{k=1}^{N_{Dj}} \lambda_{Djk}^{\mathrm{B}}(t) \widetilde{P}_{Djk}(t) - \sum_{t \in T} \sum_{i \in G} \sum_{b=1}^{N_{Gi}} \lambda_{Gib}^{\mathrm{B}}(t) \widetilde{P}_{Gib}(t), \tag{2.80}$$

where $\lambda_{Djk}^{\mathrm{B}}(t)$ is the price bid by demand j to buy power block k in hour t; and $\lambda_{Gib}^{\mathrm{B}}(t)$ is the price bid by generating unit i to sell power block b in hour t.

Note that the variables $\widetilde{P}_{Gib}(t)$ and $\widetilde{P}_{Djk}(t)$ are equal to $P_{Gib}(t)$ and $P_{Djk}(t)$, respectively, via explicit constraints. Power generation and demand variables are replicated to make the problems of the generating companies and the consumers compatible with the ISO problem. Replication of these variables permits stating the problem of the generating companies, the consumers and the ISO as a single linear complementarity problem defined by the optimality conditions of these problems. This requirement comes from the fact that a linear complementarity problem has the same number of constraints as variables. If power generation and demand variables had not been replicated, the number of constraints would have been higher than the number of variables.

2.4.3 Constraints

The ISO is responsible for physically controlling the electricity system to maintain its security and reliability. Therefore, the ISO enforces certain network constraints to ensure a secure operation. The constraints of the ISO problem are explained in the following subsections. Note that losses are not included in the formulation of these constraints but will be considered in Subsection 2.4.6.

2.4.3.1 Network Model

The transmission network plays a crucial role in the functioning of the market. A detailed model of the network implies considerable complexity in the model. Consequently, one must decide on the proper tradeoff between modeling details and computational burden.

The transmission network can be represented in detail using an AC power flow model that includes the nonlinear equations that govern power flows through the network. This model is typically used in short-term planning, reliability analysis and system operations. To achieve numerical tractability, in our study the transmission network is represented by a DC power flow.

The DC model is a linear approximation that accounts only for the active power flows (not reactive power flows) and the voltage angles (not voltage magnitudes). In this model, voltage magnitudes are assumed to be the same for all nodes and equal to one. Within this DC model both Kirchhoff current and voltage laws apply. Using these two laws, the flows within the transmission network can be uniquely identified using the power balance at every node. Besides, the power flowing through any line is limited using the available transmission capability. Further details can be found in [8, 44, 46]. Therefore, network constraints considered by the ISO are power balances at every node, capacity limits at every line and voltage angle bounds.

Note that deterministic [68] or probabilistic [13] security constraints can be incorporated into the formulation of the ISO problem. At the cost of increasing the computational burden, line and generator contingencies can also be incorporated into the model for the ISO, as stated in [81]. Moreover, transmission losses can be incorporated as stated in, for instance, [27, 30, 68, 93]. Based on numerical experience, adding losses does not significantly change results. However, in Section 2.4.6 a formulation including losses is presented.

2.4.3.2 Power Balances

At every node of the transmission network, power balance is enforced. The power balance establishes that generation injection at a node for all the units located at that node minus the demand extracted from that node for all the demands located at that node minus power reaching the adjacent nodes from the node must be equal to zero. The following expression states the power balance at every node of the network for every time period,

$$\sum_{i\in\theta_n}\sum_{b=1}^{N_{Gi}}\widetilde{P}_{Gib}(t)-\sum_{j\in\vartheta_n}\sum_{k=1}^{N_{Dj}}\widetilde{P}_{Djk}(t)-\sum_{m\in\Omega_n}B_{nm}\big[\delta_n(t)-\delta_m(t)\big]=0;$$
$$\forall n\in N;\forall t\in T, \tag{2.81}$$

where B_{nm} is the susceptance of line $n-m$ and $\delta_n(t)$ is the voltage angle of bus n in hour t.

2.4.3.3 Line Capacity Limits

For every line, the power flowing through any line (in either of the two directions) should be below a security bound which is called the line transmission capacity limit. This requirement is formulated as

$$B_{nm}\big[\delta_n(t)-\delta_m(t)\big]\le P_{nm}^{\max};\ \forall n\in N;\forall m\in\Omega_n;\forall t\in T, \tag{2.82}$$

where $P_{nm}^{\max}$ is the transmission capacity limit of line $n-m$.

2.4.3.4 Bounds on Voltage Angles

Values of voltage angles are limited to between 0 and 2π radians. These bounds are included as

$$\delta_n(t) \leq 2\pi;\ \forall n \in N; \forall t \in T \tag{2.83}$$

$$\delta_n(t) \geq 0;\ \forall n \in N; \forall t \in T. \tag{2.84}$$

2.4.3.5 Variable Replication

As explained in Section 2.4.2, power generation and demand variables are replicated to make the problems of the generating companies and the consumers compatible with the ISO problem. The following equations enforce that the power generated and demanded in the ISO problem are equal to the power generated and demanded in the problems of the generating companies and the consumers, respectively,

$$P_{Gib}(t) - \widetilde{P}_{Gib}(t) = 0;\ \forall i \in G; b = 1, \ldots, N_{Gi}; \forall t \in T \tag{2.85}$$

$$P_{Djk}(t) - \widetilde{P}_{Djk}(t) = 0;\ \forall j \in D; k = 1, \ldots, N_{Dj}; \forall t \in T. \tag{2.86}$$

2.4.4 Formulation of the ISO Problem without Losses

The ISO clears the market by seeking maximum social welfare for the whole multi-period time framework and enforcing network constraints. The ISO problem is formulated as the following linear programming problem.

Maximize

$$\sum_{t\in T}\sum_{j\in D}\sum_{k=1}^{N_{Dj}} \lambda^{\mathrm{B}}_{Djk}(t)\widetilde{P}_{Djk}(t) - \sum_{t\in T}\sum_{i\in G}\sum_{b=1}^{N_{Gi}} \lambda^{\mathrm{B}}_{Gib}(t)\widetilde{P}_{Gib}(t) \tag{2.87}$$

subject to

$$\begin{aligned} &-\sum_{i\in\theta_n}\sum_{b=1}^{N_{Gi}} \widetilde{P}_{Gib}(t) + \sum_{j\in\vartheta_n}\sum_{k=1}^{N_{Dj}} \widetilde{P}_{Djk}(t) \\ &\quad + \sum_{m\in\Omega_n} B_{nm}\big[\delta_n(t) - \delta_m(t)\big] = 0 :\ \rho_n(t);\ \forall n \in N; \forall t \in T \end{aligned} \tag{2.88}$$

$$\begin{aligned} &B_{nm}\big[\delta_n(t) - \delta_m(t)\big] \leq P^{\max}_{nm} \ :\ \gamma_{nm}(t);\ \forall n \in N; \forall m \in \Omega_n; \\ &\quad \forall t \in T \end{aligned} \tag{2.89}$$

$$P_{Gib}(t) - \widetilde{P}_{Gib}(t) = 0 :\ \mu_{Gib}(t);\ \forall i \in G; b = 1, \ldots, N_{Gi}; \forall t \in T \tag{2.90}$$

$$P_{Djk}(t) - \widetilde{P}_{Djk}(t) = 0 :\ \nu_{Djk}(t);\ \forall j \in D; k = 1, \ldots, N_{Dj}; \forall t \in T \tag{2.91}$$

$$\delta_n(t) \leq 2\pi :\ \zeta_n(t);\ \forall n \in N; \forall t \in T \tag{2.92}$$

$$\delta_n(t) \geq 0;\ \forall n \in N; \forall t \in T. \tag{2.93}$$

The objective function (2.87) is the net social welfare. It is subject to enforcing power balance at every node (2.88), line capacity limits (2.89), that the power generated and demanded in the ISO problem are equal to the power generated and demanded in the problems of the generating companies and the consumers, (2.90) and (2.91) respectively, and bounds on voltage angles, (2.92) and (2.93).

2.4.5 First Order Optimality Conditions

The ISO problem is a linear programming problem, so the KKT optimality conditions [6] are both necessary and sufficient for describing optimal points.

The optimality conditions for problem (2.87)-(2.93) are to find the generation power block levels $\widetilde{P}_{Gib}(t)$, the demand power block levels $\widetilde{P}_{Djk}(t)$, voltage angle $\delta_n(t)$, and dual variables $\rho_n(t)$, $\gamma_{nm}(t)$, $\mu_{Gib}(t)$, $\nu_{Djk}(t)$ and $\zeta_n(t)$ such that,

$$0 = \lambda^{\mathrm{B}}_{Gib}(t) - \rho_{n(i)}(t) + \mu_{Gib}(t);\ \forall i \in G; b = 1, \dots, N_{Gi}; \forall t \in T \tag{2.94}$$

$$0 = \rho_{n(j)}(t) - \lambda^{\mathrm{B}}_{Djk}(t) + \nu_{Djk}(t);\ \forall j \in D; k = 1, \dots, N_{Dj}; \forall t \in T \tag{2.95}$$

$$0 \le \sum_{m\in\Omega_n} B_{nm}\big[\rho_n(t) - \rho_m(t)\big] + \sum_{m\in\Omega_n} B_{nm}\big[\gamma_{nm}(t) - \gamma_{mn}(t)\big] + \zeta_n(t)$$
$$\perp\ \delta_n(t) \ge 0;\ \forall n \in N; \forall t \in T \tag{2.96}$$

$$0 = -\sum_{i\in\theta_n}\sum_{b=1}^{N_{Gi}} \widetilde{P}_{Gib}(t) + \sum_{j\in\vartheta_n}\sum_{k=1}^{N_{Dj}} \widetilde{P}_{Djk}(t) + \sum_{m\in\Omega_n} B_{nm}\big[\delta_n(t) - \delta_m(t)\big];$$
$$\forall n \in N; \forall t \in T \tag{2.97}$$

$$0 \le P^{\max}_{nm} - B_{nm}\big[\delta_n(t) - \delta_m(t)\big]\ \perp\ \gamma_{nm}(t) \ge 0;\ \forall n \in N;$$
$$\forall m \in \Omega_n; \forall t \in T \tag{2.98}$$

$$0 = P_{Gib}(t) - \widetilde{P}_{Gib}(t);\ \forall i \in G; b = 1, \dots, N_{Gi}; \forall t \in T \tag{2.99}$$

$$0 = P_{Djk}(t) - \widetilde{P}_{Djk}(t);\ \forall j \in D; k = 1, \dots, N_{Dj}; \forall t \in T \tag{2.100}$$

$$0 \le 2\pi - \delta_n(t)\ \perp\ \zeta_n(t) \ge 0;\ \forall n \in N; \forall t \in T. \tag{2.101}$$

Free dual variables, $\rho_n(t)$, $\mu_{Gib}(t)$ and $\nu_{Djk}(t)$, are associated with equations (2.94), (2.95), (2.97), (2.99) and (2.100), respectively.

Note that a similar simplification to that made for the problem of the generating companies has been carried out in this subsection, eliminating the dual variable of equation (2.93).

2.4.6 Formulation of the ISO Problem Considering Losses

The ISO problem is formulated including a linearized version of losses similar to the one presented in [27, 30, 68, 93], where the convexity and good computational behavior of this linearization is shown. The linearization of losses is developed below.

Under a flat voltage assumption, the real power flow through line $n-m$ computed at node n, $P_{nm}(t)$ and at node m, $P_{mn}(t)$ [8, 44, 46] are given by

$$P_{nm}(t) = G_{nm}\cos\big(\delta_n(t)-\delta_m(t)\big) + B_{nm}\sin\big(\delta_n(t)-\delta_m(t)\big) - G_{nm} \quad (2.102)$$
$$P_{mn}(t) = G_{nm}\cos\big(\delta_n(t)-\delta_m(t)\big) - B_{nm}\sin\big(\delta_n(t)-\delta_m(t)\big) - G_{nm}. \quad (2.103)$$

The losses incurred in line $n-m$ can be expressed as

$$\begin{aligned} P^{\text{loss}}_{nm}(t) &= P_{nm}(t) + P_{mn}(t) = -2G_{nm}\Big[1-\cos\big(\delta_n(t)-\delta_m(t)\big)\Big] \\ &\simeq -G_{nm}\big(\delta_{nm}(t)\big)^2, \end{aligned} \quad (2.104)$$

where G_{nm} is the conductance of line $n-m$; and $\delta_{nm}(t)$ is the voltage angle difference between nodes n and m at period t. The latter equality follows from a second-order approximation of the cosine function, which has proven to be a good approximation of the losses in a line under normal operation.

A linear approximation of the quadratic term of the right-hand side of the equation (2.104) can be obtained using L piecewise linear blocks. That is,

$$P^{\text{loss}}_{nm}(t) = -G_{nm}\sum_{l=1}^{L}\alpha_{nm,l}(t)|\delta_{nm,l}(t)|, \quad (2.105)$$

where $\delta_{nm,l}(t)$ is the l^{th} voltage angle difference block relative to nodes n and m at period t; $\alpha_{nm,l}(t)$ is the slope of the l^{th} block of voltage angle difference of nodes n and m at period t; and L is the number of blocks for the linearization of losses.

If the piecewise length is $\Delta\delta$ for all the blocks, the slope can be expressed as

$$\alpha_{nm,l}(t) = (2l-1)\Delta\delta, \quad (2.106)$$

and the absolute value function is expressed as

$$|\delta_{nm,l}(t)| = \delta^{+}_{nm,l}(t) + \delta^{-}_{nm,l}(t), \quad (2.107)$$

where $\delta^{+}_{nm,l}(t)$ and $\delta^{-}_{nm,l}(t)$ are respectively the positive and negative part of the voltage angle difference of block l between nodes n and m at period t.

With the substitution of the last expressions in the equation (2.105) we obtain

$$P^{\text{loss}}_{nm}(t) = -G_{nm}\Delta\delta\sum_{l=1}^{L}(2l-1)\big[\delta^{+}_{nm,l}(t) + \delta^{-}_{nm,l}(t)\big]. \quad (2.108)$$

This expression represents linearized losses in line $n-m$. Moreover, losses in line $n-m$ can be interpreted as additional demands at nodes n

and m, which are divided equally between nodes n and m. So, the following expression represents the losses assigned to node n,

$$P_n^{\text{loss}}(t) = -\frac{1}{2}\sum_{m\in\Omega_n}\Big[G_{nm}\Delta\delta\sum_{l=1}^{L}(2l-1)\big[\delta^+_{nm,l}(t)+\delta^-_{nm,l}(t)\big]\Big];\ \forall t\in T. \quad (2.109)$$

Next, the formulation of the ISO problem is developed including losses through equation (2.109) and considering that,

$$\delta_n(t)-\delta_m(t)=\sum_{l=1}^{L}\big(\delta^+_{nm,l}(t)-\delta^-_{nm,l}(t)\big). \quad (2.110)$$

The ISO problem considering losses is formulated as the following linear programming problem.

Maximize

$$\sum_{t\in T}\sum_{j\in D}\sum_{k=1}^{N_{Dj}}\lambda^{\text{B}}_{Djk}(t)\widetilde{P}_{Djk}(t)-\sum_{t\in T}\sum_{i\in G}\sum_{b=1}^{N_{Gi}}\lambda^{\text{B}}_{Gib}(t)\widetilde{P}_{Gib}(t) \quad (2.111)$$

subject to

$$\begin{aligned}&-\sum_{i\in\theta_n}\sum_{b=1}^{N_{Gi}}\widetilde{P}_{Gib}(t)+\sum_{j\in\vartheta_n}\sum_{k=1}^{N_{Dj}}\widetilde{P}_{Djk}(t)+\sum_{m\in\Omega_n}B_{nm}\sum_{l=1}^{L}\big[\delta^+_{nm,l}(t)-\delta^-_{nm,l}(t)\big]\\&-\frac{1}{2}\sum_{m\in\Omega_n}\Big[G_{nm}\Delta\delta\sum_{l=1}^{L}(2l-1)\big[\delta^+_{nm,l}(t)+\delta^-_{nm,l}(t)\big]\Big]=0:\ \rho_n(t);\\&\forall n\in N;\forall t\in T \qquad (2.112)\end{aligned}$$

$$\begin{aligned}&B_{nm}\sum_{l=1}^{L}\big[\delta^+_{nm,l}(t)-\delta^-_{nm,l}(t)\big]\le P^{\max}_{nm}:\ \gamma_{nm}(t);\ \forall n\in N;\\&\forall m\in\Omega_n;\forall t\in T \qquad (2.113)\end{aligned}$$

$$P_{Gib}(t)-\widetilde{P}_{Gib}(t)=0:\ \mu_{Gib}(t);\ \forall i\in G;b=1,\dots,N_{Gi};\forall t\in T \quad (2.114)$$

$$P_{Djk}(t)-\widetilde{P}_{Djk}(t)=0:\ \nu_{Djk}(t);\ \forall j\in D;k=1,\dots,N_{Dj};\forall t\in T \quad (2.115)$$

$$\delta^+_{nm,l}(t)\le\Delta\delta:\ \zeta^+_{nm,l}(t);\ \forall n\in N;\forall m\in\Omega_n;l=1,\dots,L;\forall t\in T \quad (2.116)$$

$$\delta^-_{nm,l}(t)\le\Delta\delta:\ \zeta^-_{nm,l}(t);\ \forall n\in N;\forall m\in\Omega_n;l=1,\dots,L;\forall t\in T \quad (2.117)$$

$$\delta^+_{nm,l}(t)\ge 0;\ \forall n\in N;\forall m\in\Omega_n;l=1,\dots,L;\forall t\in T \quad (2.118)$$

$$\delta^-_{nm,l}(t)\ge 0;\ \forall n\in N;\forall m\in\Omega_n;l=1,\dots,L;\forall t\in T. \quad (2.119)$$

The objective function (2.111) is the net social welfare and is the same as the case with no loss, equation (2.87). It is subject to enforcing power balance at every node and at every period (2.112), line capacity limits through every line and every time period (2.113), that the power generated and demanded in the ISO problem are equal to the power generated and demanded in the problems of the generating companies and the consumers, (2.114) and (2.115) respectively, and bounds on voltage angle difference blocks, (2.116)-(2.119).

For every node, equation (2.112) includes four terms: generation injected at the node, demand extracted from the node, power reaching the node from adjacent nodes and losses.

In the context of an optimal power flow, Schweppe et al. [80] proposes a formulation similar to (2.111)-(2.119), although nonlinear. In an equilibrium modeling context, the loss-affected piecewise formulation (2.111)-(2.119) is similar to the one used in [30].

2.4.7 First Order Optimality Conditions

The ISO problem has been formulated through a linear programming problem, therefore, the KKT optimality conditions [6] are both necessary and sufficient for describing optimal points.

The optimality conditions for problem (2.111)-(2.119) are to find the generation power block levels $\widetilde{P}_{Gib}(t)$, the demand power blocks levels $\widetilde{P}_{Djk}(t)$, the positive part of the voltage angle difference blocks $\delta^{+}_{nm,l}(t)$, the negative part of the voltage angle difference blocks $\delta^{-}_{nm,l}(t)$, and dual variables $\rho_n(t)$, $\gamma_{nm}(t)$, $\mu_{Gib}(t)$, $\nu_{Djk}(t)$, $\zeta^{+}_{nm,l}(t)$ and $\zeta^{-}_{nm,l}(t)$, such that

$$0 = \lambda^{\mathrm{B}}_{Gib}(t) - \rho_{n(i)}(t) + \mu_{Gib}(t);\ \forall i \in G; b = 1, \dots, N_{Gi}; \forall t \in T \tag{2.120}$$

$$0 = \rho_{n(j)}(t) - \lambda^{\mathrm{B}}_{Djk}(t) + \nu_{Djk}(t);\ \forall j \in D; k = 1, \dots, N_{Dj}; \forall t \in T \tag{2.121}$$

$$\begin{aligned} 0 \le\ & \rho_n(t)\big[B_{nm} - \frac{1}{2}G_{nm}\Delta\delta(2l-1)\big] + B_{nm}\,\gamma_{nm}(t) + \zeta^{+}_{nm,l}(t) \\ & \perp\ \delta^{+}_{nm,l}(t) \ge 0;\ \forall n \in N; \forall m \in \Omega_n; l = 1, \dots, L; \forall t \in T \end{aligned} \tag{2.122}$$

$$\begin{aligned} 0 \le\ & \rho_n(t)\big[-B_{nm} - \frac{1}{2}G_{nm}\Delta\delta(2l-1)\big] - B_{nm}\,\gamma_{nm}(t) + \zeta^{-}_{nm,l}(t) \\ & \perp\ \delta^{-}_{nm,l}(t) \ge 0;\ \forall n \in N; \forall m \in \Omega_n; l = 1, \dots, L; \forall t \in T \end{aligned} \tag{2.123}$$

$$\begin{aligned} 0 =\ & -\sum_{i\in\theta_n}\sum_{b=1}^{N_{Gi}} \widetilde{P}_{Gib}(t) + \sum_{j\in\vartheta_n}\sum_{k=1}^{N_{Dj}} \widetilde{P}_{Djk}(t) + \sum_{m\in\Omega_n} B_{nm}\sum_{l=1}^{L}\big[\delta^{+}_{nm,l}(t) - \delta^{-}_{nm,l}(t)\big] \\ & - \frac{1}{2}\sum_{m\in\Omega_n}\Big[G_{nm}\Delta\delta\sum_{l=1}^{L}(2l-1)\big[\delta^{+}_{nm,l}(t) + \delta^{-}_{nm,l}(t)\big]\Big]; \\ & \forall n \in N; \forall t \in T \end{aligned} \tag{2.124}$$

$$\begin{aligned} 0 \le\ & P^{\max}_{nm} - B_{nm}\sum_{l=1}^{L}\big[\delta^{+}_{nm,l}(t) - \delta^{-}_{nm,l}(t)\big]\ \perp\ \gamma_{nm}(t) \ge 0;\ \forall n \in N; \\ & \forall m \in \Omega_n; \forall t \in T \end{aligned} \tag{2.125}$$

$$0 = P_{Gib}(t) - \widetilde{P}_{Gib}(t);\ \forall i \in G; b = 1, \dots, N_{Gi}; \forall t \in T \tag{2.126}$$

$$0 = P_{Djk}(t) - \widetilde{P}_{Djk}(t);\ \forall j \in D; k = 1, \ldots, N_{Dj}; \forall t \in T \tag{2.127}$$

$$0 \leq \Delta\delta - \delta^{+}_{nm,l}(t)\ \perp\ \zeta^{+}_{nm,l}(t) \geq 0;\ \forall n \in N; \forall m \in \Omega_n; l = 1, \ldots, L;\ \forall t \in T \tag{2.128}$$

$$0 \leq \Delta\delta - \delta^{-}_{nm,l}(t)\ \perp\ \zeta^{-}_{nm,l}(t) \geq 0;\ \forall n \in N; \forall m \in \Omega_n; l = 1, \ldots, L;\ \forall t \in T. \tag{2.129}$$

Free dual variables, $\rho_n(t)$, $\mu_{Gib}(t)$ and $\nu_{Djk}(t)$, are associated with equations (2.120), (2.121), (2.124), (2.126) and (2.127), respectively.

Note that a similar simplification to that made for the problem of the generating companies has been carried out in this subsection, that is, eliminating the dual variables of equations (2.118) and (2.119).

2.5 Summary

This chapter presents the general formulation of the problems faced by each market agent in the pool as linear optimization programs. The generating companies seek to maximize their respective profits, subject to technical limits on production such as the capacity limit, the minimum power output and available maximum and minimum power output taking into account the start-up and shut-down ramp limits, and the ramp-up and ramp-down limits. The goals of the consumers are to maximize their respective economic utilities according to limits on demand such as minimum demand requirements. And finally, the ISO clears the market by taking into account network constraints and seeking maximum social welfare. Network constraints considered by the ISO are power balance at every node, capacity limit at every line and voltage angle bounds. The type of market under consideration uses the concept of locational marginal pricing, and these locational marginal prices are determined as the dual variables associated to the power balance constraints as part of the problem faced by the ISO. For the sake of clarity, the problem of the ISO is formulated with and without a consideration of network losses. Once these linear programming problems are formulated, we obtain their respective KKT optimality conditions that describe their optimal points.

Chapter 3

Single-Period Equilibrium / Near-Equilibrium

3.1 Introduction

This chapter provides a tool to find the equilibrium of an electricity market in a single period of time, typically one hour. The market includes generating companies and consumers as well as an Independent System Operator (ISO) [56, 81, 82]. Using this tool, market agents simultaneously optimize their respective individual and conflicting objectives.

The market presents a pool format in which each generating company bids a set of energy production blocks and their corresponding minimum selling prices, and each consumer bids a set of consumption energy blocks and their corresponding maximum buying prices. In turn, the ISO clears the market seeking maximum social welfare. So, the single-period equilibrium is computed once the bidding stacks of every unit of each generating company and the bidding stacks of every demand of each consumer are known.

The equilibrium is defined as the energy transaction levels and their associated prices that result in:

- maximum profit for every individual generating company
- maximum utility for every individual consumer
- maximum net social welfare for the ISO.

The definition of a single-period equilibrium is illustrated in Figure 3.1.

We consider a detailed network representation of the electric power system, consisting of a linearized power flow model that includes a precise representation of losses [27, 30, 68, 93]. This linear model permits the computing of locational marginal prices while taking into account the effects of line congestion and transmission losses in an accurate and efficient manner. The

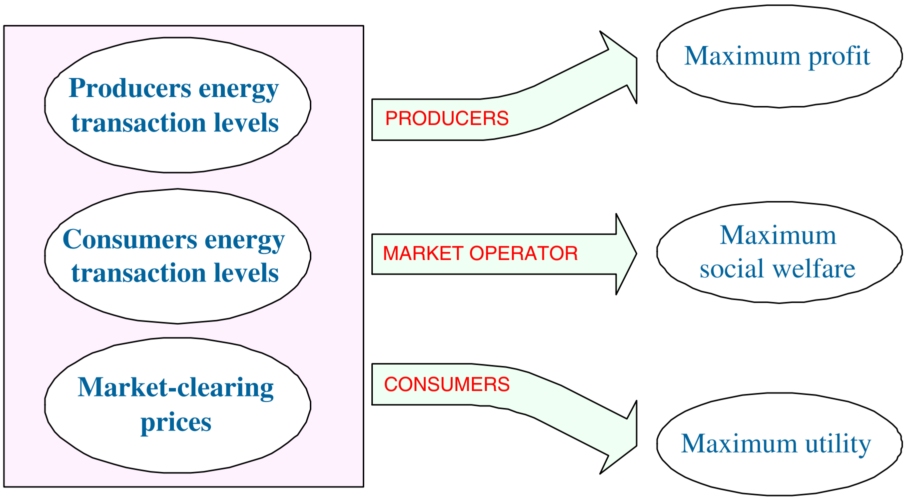

Figure 3.1: Definition of single-period equilibrium

demands are taken to be non-constant and their values are determined as part of the solution.

The generating companies and the consumers are modeled as solving appropriate optimization problems with locational marginal prices as inputs. The locational marginal prices are determined as dual prices to balance constraints which are part of an optimization problem faced by the ISO. Using a linear model to represent the network, each of these three sets of optimization problems are linear programming problems. Thus, the Karush-Kuhn-Tucker (KKT) optimality conditions [6] are both necessary and sufficient for describing optimal points. The optimality conditions of the three sets of problems result in a Mixed Linear Complementarity Problem (MLCP) that can be solved as the optimal solution of an equivalent Quadratic Programming Problem (QPP) [25, 59].

Conditions that ensure minimum profit for the generating units, which are relevant in actual markets and represent an important modeling advantage, can be included. These conditions may render a generating unit uncompetitive and expel it from the market. The minimum profit conditions can be included as additional constraints to the equivalent quadratic problem mentioned previously.

This chapter provides the formulation of the equilibrium problem in both cases, not considering and considering minimum profit conditions, and efficient solution techniques to solve these problems are developed. In addition, the equilibrium problem is compared with an optimal power flow. It should be noted that unlike the optimal power flow, the proposed equilibrium procedure allows incorporating minimum profit conditions for on-line generating units.

3.2 Equilibrium without Minimum Profit Conditions

For the single-period case, equilibrium can be obtained considering the set of continuous optimization problems corresponding to the maximum profit / utility of the generating companies / consumers and the maximum social welfare of the independent system operator, and corresponding optimality conditions [6] result in a mixed linear complementary problem which is easy to solve [25, 50, 59]. The solution of this MLCP provides a market equilibrium.

This section formulates the single-period equilibrium and proposes a solution technique to determine the market equilibrium. This market equilibrium is compared with the solution of an optimal power flow and an example is used to illustrate the results of this comparison.

3.2.1 Problem Formulation

An equilibrium optimizes individual and conflicting objectives of each market agent simultaneously. The market participants considered in this work are generating companies, consumers and an independent system operator. The models of market participants have been explained in Chapter 2 in a general manner. Below, these models are defined for a single-period case.

In this chapter, the considered time horizon is therefore one time period. For this reason, the time variable t is eliminated.

Additional simplifications are made in the models of the generating companies. The model for a generating company developed in Chapter 2 (equations (2.22)-(2.32)) includes constraints that link one period with the following and the previous one, such as ramp rate limits. These constraints are not relevant in the single-period case, so they are not taken into account in modeling generating companies to obtain the single-period market equilibrium. Note that fixed, start-up and shut-down costs should be considered in a multi-period framework, but not in the single-period case, so these costs are also eliminated from the formulation. The last remark is that minimum power output of every generating unit is equal to zero in order to avoid the use of binary variables in the formulation of the problem. Avoiding the use of binary variables in the formulation allows the use of the Karush-Kuhn-Tucker conditions to derive the optimal solution of the problem.

If the above simplifications are not made, the single-period problem can be solved using the technique developed in Chapter 4 to solve the multi-period problem.

In the equilibrium, the generating companies maximize their respective profits. The problem of any generating company can be formulated as the following linear programming problem, maximize

$$\sum_{i \in G_f} \sum_{b=1}^{N_{Gi}} \big(\rho_{n(i)} - \lambda^{\mathrm{C}}_{Gib}\big) P_{Gib} \tag{3.1}$$

subject to

$$\sum_{b=1}^{N_{Gi}} P_{Gib} \le P^{\max}_{Gi} : \ \alpha_i; \ \forall i \in G_f \tag{3.2}$$

$$P_{Gib} \le P^{\max}_{Gib} : \ \phi_{ib}; \ \forall i \in G_f; b = 1, \ldots, N_{Gi} - 1 \tag{3.3}$$

$$P_{Gib} \ge 0; \ \forall i \in G_f; b = 1, \ldots, N_{Gi}. \tag{3.4}$$

The decision variables of this problem are the amounts of power to be generated by each unit i in each block b, i.e., P_{Gib}; and prices $\rho_{n(i)}$ are fixed values for the generating companies but variables in the larger overall single-period equilibrium problem.

The objective function (3.1) is the total profit for the generating company. The set of constraints (3.2) specifies the capacity limit of each generating unit and (3.3) represents the maximum capacity limit for each block of each unit, except the last block of each unit in order to avoid redundancy with constraint (3.2). Nonnegative levels of power to be generated by each unit in each block are stated by (3.4).

In what follows, the formulation of the problem of any consumer is presented. This model is similar to the one presented in Chapter 2 (equations (2.70)-(2.73)), but does not consider time variables.

Maximize

$$\sum_{j \in D_q} \sum_{k=1}^{N_{Dj}} \big(\lambda^{\mathrm{U}}_{Djk} - \rho_{n(j)}\big) P_{Djk} \tag{3.5}$$

subject to

$$\sum_{k=1}^{N_{Dj}} P_{Djk} \ge P^{\min}_{Dj} : \ \sigma_j; \ \forall j \in D_q \tag{3.6}$$

$$P_{Djk} \le P^{\max}_{Djk} : \ \varphi_{jk}; \ \forall j \in D_q; k = 1, \ldots, N_{Dj} \tag{3.7}$$

$$P_{Djk} \ge 0; \ \forall j \in D_q; k = 1, \ldots, N_{Dj}. \tag{3.8}$$

The decision variables of this problem are the amounts of power to be consumed by each demand j in each block k, i.e., P_{Djk}; and prices $\rho_{n(j)}$ are fixed values for the consumer but variables in the larger overall single-period equilibrium problem.

The objective function (3.5) represents the economic utility for the consumer. Equation (3.6) is the minimum demand requirement of each demand of the consumer, and equations (3.7)and (3.8) limit the power consumed in each block of each demand.

Lastly, the problem of the ISO is formulated including line losses and line capacity limits as problem (2.111)-(2.119) in Chapter 2, but considering only one time period.

Maximize

$$\sum_{j \in D} \sum_{k=1}^{N_{Dj}} \lambda_{Djk}^{\mathrm{B}} \widetilde{P}_{Djk} - \sum_{i \in G} \sum_{b=1}^{N_{Gi}} \lambda_{Gib}^{\mathrm{B}} \widetilde{P}_{Gib} \tag{3.9}$$

subject to

$$\begin{aligned} & - \sum_{i \in \theta_n} \sum_{b=1}^{N_{Gi}} \widetilde{P}_{Gib} + \sum_{j \in \vartheta_n} \sum_{k=1}^{N_{Dj}} \widetilde{P}_{Djk} + \sum_{m \in \Omega_n} B_{nm} \sum_{l=1}^{L} \left(\delta_{nm,l}^{+} - \delta_{nm,l}^{-} \right) \\ & - \frac{1}{2} \sum_{m \in \Omega_n} \left[G_{nm} \Delta\delta \sum_{l=1}^{L} (2l-1) \left(\delta_{nm,l}^{+} + \delta_{nm,l}^{-} \right) \right] = 0 : \ \rho_n; \ \forall n \in N \end{aligned} \tag{3.10}$$

$$B_{nm} \sum_{l=1}^{L} \left(\delta_{nm,l}^{+} - \delta_{nm,l}^{-} \right) \le P_{nm}^{\max} : \ \gamma_{nm}; \ \forall n \in N; \forall m \in \Omega_n \tag{3.11}$$

$$P_{Gib} - \widetilde{P}_{Gib} = 0 : \ \mu_{Gib}; \ \forall i \in G; b = 1, \ldots, N_{Gi} \tag{3.12}$$

$$P_{Djk} - \widetilde{P}_{Djk} = 0 : \ \nu_{Djk}; \ \forall j \in D; k = 1, \ldots, N_{Dj} \tag{3.13}$$

$$\delta_{nm,l}^{+} \le \Delta\delta : \ \zeta_{nm,l}^{+}; \ \forall n \in N; \forall m \in \Omega_n; l = 1, \ldots, L \tag{3.14}$$

$$\delta_{nm,l}^{-} \le \Delta\delta : \ \zeta_{nm,l}^{-}; \ \forall n \in N; \forall m \in \Omega_n; l = 1, \ldots, L \tag{3.15}$$

$$\delta_{nm,l}^{+} \ge 0; \ \forall n \in N; \forall m \in \Omega_n; l = 1, \ldots, L \tag{3.16}$$

$$\delta_{nm,l}^{-} \ge 0; \ \forall n \in N; \forall m \in \Omega_n; l = 1, \ldots, L. \tag{3.17}$$

The decision variables of this problem are the amounts of power to be generated by each generating unit i in each block b, i.e., $\widetilde{P}_{Gib}$; the amounts of power to be consumed by each demand j in each block k, i.e., $\widetilde{P}_{Djk}$; positive and negative part of the voltage angles difference blocks, $\delta_{nm,l}^{+}$ and $\delta_{nm,l}^{-}$; and the locational marginal prices, ρ_n.

The objective function (3.9) is the social welfare and is maximized subject to power balance at every node (3.10), line capacity limits (3.11), equality between power generated and demanded in the ISO problem and in the problems of the generating companies and the consumers, (3.12) and (3.13) respectively, as well as limits for the voltage angles difference blocks, (3.14)-(3.17).

3.2.2 Solution Technique: Mixed Linear Complementarity Problem

The market equilibrium is defined as the energy transaction levels for which the generating companies maximize their respective profits and the consumers maximize their respective utilities and the ISO maximizes the social welfare. So, we have different problems with conflicting objectives that must be optimized simultaneously.

Note that the KKT optimality conditions are both necessary and sufficient for describing the optimal point of a linear programming problem. Then, the optimality conditions of the problems of the generating companies define the optimal point of these problems, and the same applies for the optimality conditions of the problems of the consumers and the ISO problem. For this reason, the equilibrium can be defined by the optimality conditions for the problem of each generating company, each consumer and the ISO. It should be noted that we use optimality conditions (complementarity theory) to be able to include constraints on prices, i.e., on dual variables. This feature enhances the capabilities of the model we propose. For instance, it makes it possible to include minimum profit conditions for the generating companies or maximum cost conditions for the consumers. Note that it is impossible to consider such conditions when an optimal power flow or a conventional market-clearing procedure is applied because prices are not available to impose constraints on them, since they are outputs of the procedure.

Note that the KKT conditions for the problem of any generating company have the same structure as for the other generating companies, therefore, we formulate jointly the KKT optimality conditions of all the generating companies. The same reasoning is applied to the optimality conditions of all the consumers. Equations (3.18)-(3.20), which comprise the optimality conditions for the problems of all the generating companies, equations (3.21)-(3.23), which comprise the KKT conditions for the problems of all the consumers, and equations (3.24)-(3.33), the optimality conditions for the problem of the ISO, result in a mixed linear complementarity problem [25] to be solved in order to determine the market equilibrium. A mixed linear complementarity problem consists of a linear complementarity problem and a system of linear equations. In Appendix A the linear complementarity problem is described in detail.

The mixed linear complementarity problem that defines the market equilibrium can be written in compact form as,

$$0 \leq \lambda^{\mathrm{C}}_{Gib} - \rho_{n(i)} + \alpha_i + \phi_{ib} \ \perp \ P_{Gib} \geq 0; \ \forall i \in G; b = 1, \ldots, N_{Gi} \tag{3.18}$$

$$0 \leq P^{\max}_{Gi} - \sum_{b=1}^{N_{Gi}} P_{Gib} \ \perp \ \alpha_i \geq 0; \ \forall i \in G \tag{3.19}$$

$$0 \leq P^{\max}_{Gib} - P_{Gib} \ \perp \ \phi_{ib} \geq 0; \ \forall i \in G; b = 1, \ldots, N_{Gi} - 1 \tag{3.20}$$

$$0 \leq \rho_{n(j)} - \lambda^{\mathrm{U}}_{Djk} - \sigma_j + \varphi_{jk} \ \perp \ P_{Djk} \geq 0; \ \forall j \in D; k = 1, \ldots, N_{Dj} \tag{3.21}$$

$$0 \leq \sum_{k=1}^{N_{Dj}} P_{Djk} - P^{\min}_{Dj} \ \perp \ \sigma_j \geq 0; \ \forall j \in D \tag{3.22}$$

$$0 \leq P^{\max}_{Djk} - P_{Djk} \ \perp \ \varphi_{jk} \geq 0; \ \forall j \in D; k = 1, \ldots, N_{Dj} \tag{3.23}$$

$$0 = \lambda^{\mathrm{B}}_{Gib} - \rho_{n(i)} + \mu_{Gib}; \ \forall i \in G; b = 1, \ldots, N_{Gi} \tag{3.24}$$

$$0 = \rho_{n(j)} - \lambda^{\mathrm{B}}_{Djk} + \nu_{Djk};\ \forall j \in D; k = 1, \ldots, N_{Dj} \tag{3.25}$$

$$0 \le \rho_n \big[B_{nm} - \frac{1}{2} G_{nm} \Delta\delta(2l-1)\big] + B_{nm}\,\gamma_{nm} + \zeta^{+}_{nm,l} \ \perp\ \delta^{+}_{nm,l} \ge 0;$$
$$\forall n \in N; \forall m \in \Omega_n; l = 1, \ldots, L \tag{3.26}$$

$$0 \le \rho_n \big[-B_{nm} - \frac{1}{2} G_{nm} \Delta\delta(2l-1)\big] - B_{nm}\,\gamma_{nm} + \zeta^{-}_{nm,l} \ \perp\ \delta^{-}_{nm,l} \ge 0;$$
$$\forall n \in N; \forall m \in \Omega_n; l = 1, \ldots, L \tag{3.27}$$

$$0 = -\sum_{i \in \theta_n} \sum_{b=1}^{N_{Gi}} \widetilde{P}_{Gib} + \sum_{j \in \vartheta_n} \sum_{k=1}^{N_{Dj}} \widetilde{P}_{Djk} + \sum_{m \in \Omega_n} B_{nm} \sum_{l=1}^{L} \big(\delta^{+}_{nm,l} - \delta^{-}_{nm,l}\big)$$
$$- \frac{1}{2} \sum_{m \in \Omega_n} \Big[G_{nm} \Delta\delta \sum_{l=1}^{L} (2l-1) \big(\delta^{+}_{nm,l} + \delta^{-}_{nm,l}\big)\Big];\ \forall n \in N \tag{3.28}$$

$$0 \le P^{\max}_{nm} - B_{nm} \sum_{l=1}^{L} \big(\delta^{+}_{nm,l} - \delta^{-}_{nm,l}\big) \ \perp\ \gamma_{nm} \ge 0;\ \forall n \in N;$$
$$\forall m \in \Omega_n \tag{3.29}$$

$$0 = P_{Gib} - \widetilde{P}_{Gib};\ \forall i \in G; b = 1, \ldots, N_{Gi} \tag{3.30}$$

$$0 = P_{Djk} - \widetilde{P}_{Djk};\ \forall j \in D; k = 1, \ldots, N_{Dj} \tag{3.31}$$

$$0 \le \Delta\delta - \delta^{+}_{nm,l} \ \perp\ \zeta^{+}_{nm,l} \ge 0;\ \forall n \in N; \forall m \in \Omega_n; l = 1, \ldots, L \tag{3.32}$$

$$0 \le \Delta\delta - \delta^{-}_{nm,l} \ \perp\ \zeta^{-}_{nm,l} \ge 0;\ \forall n \in N; \forall m \in \Omega_n; l = 1, \ldots, L. \tag{3.33}$$

The above mixed linear complementarity formulation is a useful tool for various market participants. In particular, the market regulator benefits by identifying market equilibria as part of its market monitoring. Additionally, this market model can assist the generating companies and the consumers in their respective bidding planning processes.

Numerical simulations using different power systems show the existence of the solution; however, no formal proof of its existence has been developed.

3.2.3 Problem Size

The size of this mixed linear complementarity problem (3.18)-(3.33) is illustrated in Table 3.1. In this table, N_G and N_D represent the total number of the units and of the demands in the system, respectively; N_{GB} and N_{DK} represent the total number of blocks bid by all units and demanded by all demands, respectively; N_N and N_L represent the total number of nodes and lines of the system, respectively; and L represents the number of blocks used to linearize losses.

There are several computational methods for solving mixed linear complementarity problems (see for example [24, 25, 28, 32, 35]). Available commercial solvers include, among others, PATH [29], MILES [78] and SMOOTH

Table 3.1: Size of the MLCP

Number of positive continuous variables	Number of free continuous variables	Number of equations
$2(N_{GB}+N_{DK})+2N_L$ $+N_G+N_D+8N_LL$	$2(N_{GB}+N_{DK})+N_N$	$8(N_{GB}+N_{DK})+N_N$ $+6N_L+3N_D+24N_LL$

[19], which can be used GAMS [14], AMPL [36] or AIMMS [75]. A comparison of these commercial solvers can be found in [10].

3.2.4 Comparison with an Optimal Power Flow

The Optimal Power Flow (OPF) [15, 54, 91] is defined as an optimization problem in which certain variables are adjusted to optimize an objective function, while satisfying physical and operational constraints. The objective function is to maximize the social welfare. Constraints are basically generating power output limits, power balance at every node and real power flow limits in every line.

Unlike the optimal power flow, the equilibrium previously described allows modeling the activities of the various market participants, and more importantly, allows including constraints on prices (dual variables), such as minimum profit conditions.

A comparison between the single-period equilibrium and the optimal power flow is made in this section.

As stated in Subsection 3.2.1, the minimum power output of each generating unit is considered equal to zero, and line transmission capacity limits and a linearized version of losses are included in the model of the network.

The optimal power flow is described by the following linear programming problem.

Maximize

$$\sum_{j\in D}\sum_{k=1}^{N_{Dj}}\lambda^{\mathrm{B}}_{Djk}P_{Djk}-\sum_{i\in G}\sum_{b=1}^{N_{Gi}}\lambda^{\mathrm{B}}_{Gib}P_{Gib} \tag{3.34}$$

subject to

$$\begin{aligned}&-\sum_{i\in\theta_n}\sum_{b=1}^{N_{Gi}}P_{Gib}+\sum_{j\in\vartheta_n}\sum_{k=1}^{N_{Dj}}P_{Djk}+\sum_{m\in\Omega_n}B_{nm}\sum_{l=1}^{L}\left(\delta^+_{nm,l}-\delta^-_{nm,l}\right)\\&\quad+\frac{1}{2}\sum_{m\in\Omega_n}\left[G_{nm}\Delta\delta\sum_{l=1}^{L}(2l-1)\left(\delta^+_{nm,l}+\delta^-_{nm,l}\right)\right]=0:\ \rho_n;\ \forall n\in N\end{aligned} \tag{3.35}$$

$$B_{nm}\sum_{l=1}^{L}\left(\delta^+_{nm,l}-\delta^-_{nm,l}\right)\le P^{\max}_{nm}:\ \gamma_{nm};\ \forall n\in N;\forall m\in\Omega_n \tag{3.36}$$

$$\sum_{b=1}^{N_{Gi}} P_{Gib} \leq P_{Gi}^{\max} : \ \alpha_i; \ \forall i \in G \tag{3.37}$$

$$P_{Gib} \leq P_{Gib}^{\max} : \ \phi_{ib}; \ \forall i \in G; b = 1, \ldots, N_{Gi} - 1 \tag{3.38}$$

$$P_{Gib} \geq 0; \ \forall i \in G; b = 1, \ldots, N_{Gi} \tag{3.39}$$

$$\sum_{k=1}^{N_{Dj}} P_{Djk} \geq P_{Dj}^{\min} : \ \sigma_j; \ \forall j \in D \tag{3.40}$$

$$P_{Djk} \leq P_{Djk}^{\max} : \ \varphi_{jk}; \ \forall j \in D; k = 1, \ldots, N_{Dj} \tag{3.41}$$

$$P_{Djk} \geq 0; \ \forall j \in D; k = 1, \ldots, N_{Dj} \tag{3.42}$$

$$\delta_{nm,l}^{+} \leq \Delta\delta : \ \zeta_{nm,l}^{+}; \ \forall n \in N; \forall m \in \Omega_n; l = 1, \ldots, L \tag{3.43}$$

$$\delta_{nm,l}^{-} \leq \Delta\delta : \ \zeta_{nm,l}^{-}; \ \forall n \in N; \forall m \in \Omega_n; l = 1, \ldots, L \tag{3.44}$$

$$\delta_{nm,l}^{+} \geq 0; \ \forall n \in N; \forall m \in \Omega_n; l = 1, \ldots, L \tag{3.45}$$

$$\delta_{nm,l}^{-} \geq 0; \ \forall n \in N; \forall m \in \Omega_n; l = 1, \ldots, L. \tag{3.46}$$

The objective function (3.34) represents the social welfare that is maximized subject to technical constraints. Equation (3.35) is the power balance in each node and equation (3.36) imposes line capacity limits. Constraint (3.37) is the maximum power output of each generating unit and equations (3.38) and (3.39) represent the power bound of each production block. Equation (3.40) enforces the minimum demand requirement for each demand and equations (3.41) and (3.42) are the power limits for the consumption blocks. Finally, equations (3.43)-(3.46) represent the limits of the block modeling voltage angle differences.

The KKT optimality conditions of this optimal power flow are derived to compare them with the mixed linear complementarity problem that defines the single-period equilibrium.

$$0 \leq \lambda_{Gib}^{\mathrm{B}} - \rho_{n(i)} + \alpha_i + \phi_{ib} \ \perp \ P_{Gib} \geq 0; \ \forall i \in G; b = 1, \ldots, N_{Gi} \tag{3.47}$$

$$0 \leq P_{Gi}^{\max} - \sum_{b=1}^{N_{Gi}} P_{Gib} \ \perp \ \alpha_i \geq 0; \ \forall i \in G \tag{3.48}$$

$$0 \leq P_{Gib}^{\max} - P_{Gib} \ \perp \ \phi_{ib} \geq 0; \ \forall i \in G; b = 1, \ldots, N_{Gi} - 1 \tag{3.49}$$

$$0 \leq -\lambda_{Djk}^{\mathrm{B}} + \rho_{n(j)} - \sigma_j + \varphi_{jk} \ \perp \ P_{Djk} \geq 0; \ \forall j \in D; k = 1, \ldots, N_{Dj} \tag{3.50}$$

$$0 \leq \sum_{k=1}^{N_{Dj}} P_{Djk} - P_{Dj}^{\min} \ \perp \ \sigma_j \geq 0; \ \forall j \in D \tag{3.51}$$

$$0 \leq P_{Djk}^{\max} - P_{Djk} \ \perp \ \varphi_{jk} \geq 0; \ \forall j \in D; k = 1, \ldots, N_{Dj} \tag{3.52}$$

$$0 \leq \rho_n \big[B_{nm} - \frac{1}{2} G_{nm} \Delta\delta (2l - 1) \big] + B_{nm} \, \gamma_{nm} + \zeta_{nm,l}^{+} \ \perp \ \delta_{nm,l}^{+} \geq 0; \\ \forall n \in N; \forall m \in \Omega_n; l = 1, \ldots, L \tag{3.53}$$

$$0 \leq \rho_n\big[-B_{nm} - \frac{1}{2}G_{nm}\Delta\delta(2l-1)\big] - B_{nm}\,\gamma_{nm} + \zeta^-_{nm,l} \;\perp\; \delta^-_{nm,l} \geq 0;$$
$$\forall n \in N; \forall m \in \Omega_n; l = 1,\ldots,L \tag{3.54}$$

$$0 = -\sum_{i\in\theta_n}\sum_{b=1}^{N_{Gi}} P_{Gib} + \sum_{j\in\vartheta_n}\sum_{k=1}^{N_{Dj}} P_{Djk} + \sum_{m\in\Omega_n} B_{nm}\sum_{l=1}^{L}\big(\delta^+_{nm,l} - \delta^-_{nm,l}\big)$$
$$-\frac{1}{2}\sum_{m\in\Omega_n}\Big[G_{nm}\Delta\delta\sum_{l=1}^{L}(2l-1)\big(\delta^+_{nm,l} + \delta^-_{nm,l}\big)\Big];\; \forall n \in N \tag{3.55}$$

$$0 \leq P^{\max}_{nm} - B_{nm}\sum_{l=1}^{L}\big(\delta^+_{nm,l} - \delta^-_{nm,l}\big) \;\perp\; \gamma_{nm} \geq 0;\; \forall n \in N; \forall m \in \Omega_n \tag{3.56}$$

$$0 \leq \Delta\delta - \delta^+_{nm,l} \;\perp\; \zeta^+_{nm,l} \geq 0;\; \forall n \in N; \forall m \in \Omega_n; l = 1,\ldots,L \tag{3.57}$$

$$0 \leq \Delta\delta - \delta^-_{nm,l} \;\perp\; \zeta^-_{nm,l} \geq 0;\; \forall n \in N; \forall m \in \Omega_n; l = 1,\ldots,L. \tag{3.58}$$

Equations (3.47)-(3.58) are the KKT conditions of the optimal power flow and define a mixed linear complementarity problem.

Note that optimality conditions (3.47)-(3.52) of the optimal power flow exactly correspond to optimality conditions (3.18)-(3.23) of the equilibrium if the generating companies and the consumers are supposed to bid at their marginal cost and utility, respectively. Moreover, optimality conditions (3.53)-(3.58) of the OPF correspond to conditions (3.26)-(3.29) and (3.32)-(3.33) of the equilibrium if conditions (3.30)-(3.31) are included in the power balance equation (3.28). So, the only conditions of the equilibrium not considered by the optimality conditions of the optimal power flow are conditions (3.24) and (3.25), which do not enforce any restriction on prices because variables μ_{Gib} and ν_{Djk} can take any value. Therefore, the single-period equilibrium is equivalent to the optimal power flow if price bids of the generating companies and the consumers correspond to their marginal costs and utilities, respectively, and if no minimum profit conditions for the generating units are considered.

3.2.5 Example

This example has been designed to illustrate the single-period equilibrium and to verify the equivalence between the equilibrium without minimum profit conditions and the optimal power flow.

3.2.5.1 Data

The 4-node network [46] depicted in Figure 3.2 is used in this numerical example. The network includes two generating units located at nodes 1 and 2, respectively. The maximum and minimum power output of each generating unit is shown in the figure. There are power consumptions at nodes 3 and 4, and their respective minimum demand requirements are indicated in Figure

3.2. The generating units belong to two different generating companies, and the demands to two different consumers, but ownership does not affect results.

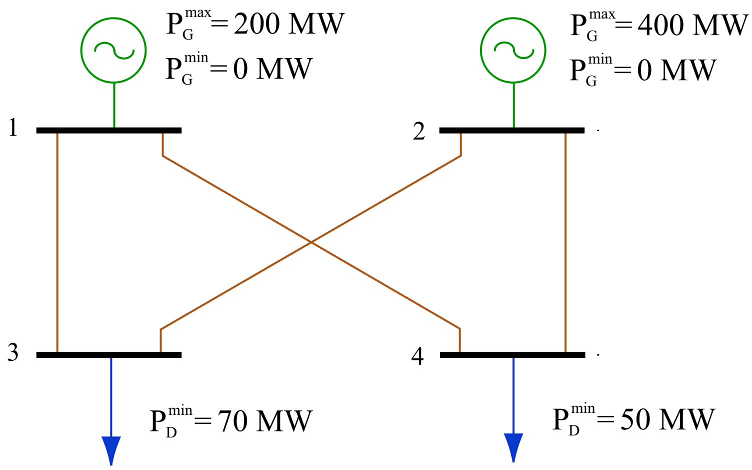

Figure 3.2: 4-node network. Example 3.2.5

Every generating unit bids at its marginal costs using two blocks. The size and price of each block of each generating unit are shown in Table 3.2.

Table 3.2: Generating unit bids. Example 3.2.5

Unit	Block 1		Block 2	
	Size [MW]	Price [$/MWh]	Size [MW]	Price [$/MWh]
1	100	18.6	100	20.3
2	150	19.2	250	20.0

Every demand uses two blocks to bid on the market and prices associated with these blocks correspond to their respective utilities. Table 3.3 shows the size and the price of each block bid by each demand.

Line data can be found in Table 3.4 and include the value of the line parameters such as resistance, reactance and shunt susceptance, and the line capacity limits. The line parameters are per unit (three-phase base of 230 kV and 100 MVA).

The bus admittance matrix [8] is shown below. This is obtained using the line parameter provided in Table 3.4.

Table 3.3: Demand bids. Example 3.2.5

Node	Block 1		Block 2	
	Size [MW]	Price [$/MWh]	Size [MW]	Price [$/MWh]
3	120	23.5	130	20.5
4	100	22.0	150	19.0

Table 3.4: Line data. Example 3.2.5

From node	To node	R [pu]	X [pu]	B [pu]	Capacity limit [MW]
1	3	0.01008	0.0504	0.1025	100
1	4	0.00744	0.0372	0.0775	100
2	3	0.00744	0.0372	0.0775	150
2	4	0.01272	0.0636	0.1275	200

$$Y_{\text{bus}} = \begin{pmatrix} 8.98 - 44.84i & 0 & -3.82 + 19.08i & -5.17 + 25.85i \\ 0 & 8.19 - 40.92i & -5.17 + 25.85i & -3.02 + 15.12i \\ -3.82 + 19.08i & -5.17 + 25.85i & 8.98 - 44.84i & 0 \\ -5.17 + 25.85i & -3.02 + 15.12i & 0 & 0 \end{pmatrix}$$

The real part of the bus matrix is represented by G and the imaginary part by B. That is,

$$Y_{\text{bus}} = G + iB. \tag{3.59}$$

Finally, four blocks have been used to linearize losses and the piecewise length for all blocks is $\Delta\delta = 5^{\text{o}} = 0.087$ rad.

3.2.5.2 Equilibrium Problem

Figure 3.3 shows the demand curve and the supply curve of the example 3.2.5.

Table 3.5 provides results of the single-period equilibrium concerning generating unit output, revenues and profits. These results have been obtained by directly solving the mixed linear complementarity problem (3.18)-(3.33).

Table 3.6 shows the power consumed and the corresponding demand costs.

Power flows and losses through each line are provided in Table 3.7, as well as voltage angle difference blocks. Note that the power flowing through line 2-3 is the same as its capacity limit, so this line is working at its maximum capacity.

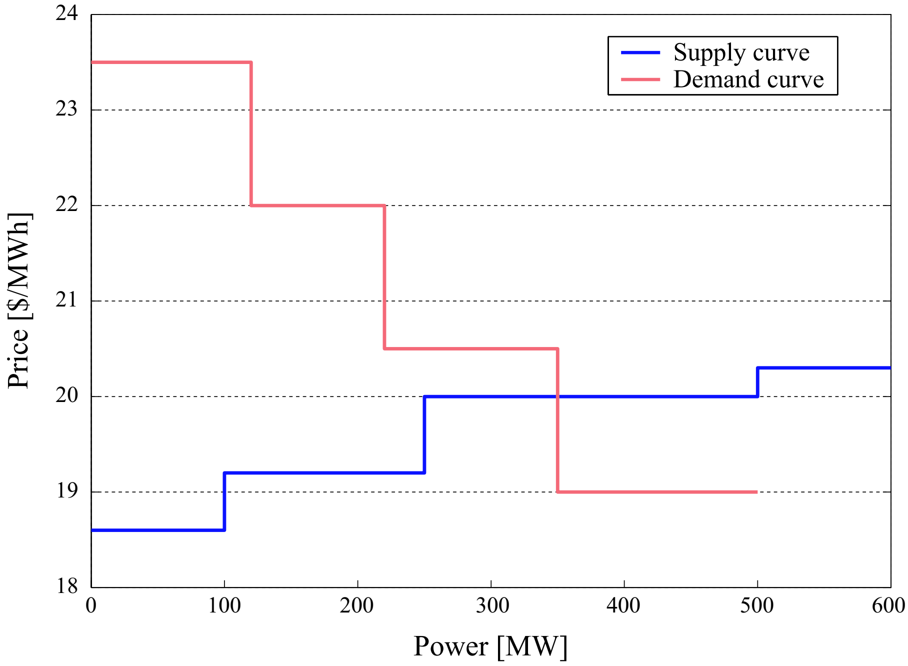

Figure 3.3: Demand and supply curves for the example 3.2.5

Table 3.5: Results for the generating units. Example 3.2.5

Unit	Power output [MW]	Revenue [$/h]	Profit [$/h]
1	100.00	2023.34	163.34
2	217.15	4343.06	120.00
Total	317.15	6366.40	283.34

Table 3.6: Results for the demands. Example 3.2.5

Node	Power consumed [MW]	Demand cost [$/h]
3	214.14	4389.86
4	100.00	2032.19
Total	314.14	6422.05

The equilibrium provides locational marginal prices for all nodes, which are shown in Table 3.8. Prices are different across nodes due to congestion in line 2-3.

Table 3.7: Results for lines. Example 3.2.5

From node	To node	δ^+_{nm} [rad]	δ^-_{nm} [rad]	Power flow [MW]	Losses [MW]
1	3	0.0342	0.0000	65.28	0.4913
1	4	0.0133	0.0000	34.40	0.1501
2	3	0.0580	0.0000	150.00	1.7962
2	4	0.0436	0.0000	65.97	0.5757

Table 3.8: Locational marginal prices. Example 3.2.5

Node	Locational marginal price [$/MWh]
1	20.2334
2	20.0000
3	20.5000
4	20.3219

3.2.5.3 Comparison with the OPF

The optimal power flow of the 4-node network is obtained by solving the linear programming problem (3.34)-(3.46). The results are exactly the same as those shown in the previous subsection, obtained through the single-period equilibrium.

3.3 Equilibrium Including Minimum Profit Conditions

This section analyzes the single-period equilibrium of a pool-based electricity market working under locational marginal pricing that includes minimum profit conditions for on-line generating units. This equilibrium is obtained through the solution of a quadratic programming problem equivalent to the mixed linear complementarity problem defined in Subsection 3.2.2, including the minimum profit conditions.

Contrary to what may happen if an optimal power flow approach is used, the proposed procedure includes constraints that force any generating unit that operates to meet a pre-specified minimum profit. These conditions may render a generating unit uncompetitive and expel it from the market.

To include such minimum profit conditions in the single-period equilibrium problem can originate infeasibilities in the solution of the problem. In this case, we said we have achieved a near-equilibrium. This behavior is explained in detail in this section.

The formulation of the single-period equilibrium that defines the equilibrium under minimum profits conditions is analyzed in this section and the effect of imposing these conditions on the market equilibrium is discussed. Then, different techniques to solve the equilibrium are presented and are illustrated using the 4-node network.

3.3.1 Justifications for Minimum Profit Conditions

Minimum profit requirements are similar to conditions used in some actual markets [73] to ensure peak profitability and to promote generation capacity investment. Apart from that, minimum profit condition can be used to internalize fixed and other costs that do not directly appear in the bidding stack.

It is important to discuss these minimum profit conditions in rather more depth since they are computationally challenging yet realistic aspects of electricity markets. For clarity, we describe a equilibrium procedure similar to the one solved in the pool-based electricity market of mainland Spain, which uses these minimum profit conditions. Hours are considered one at a time and a simple auction mechanism is used to identify the clearing price for each hour as the intersection of the stepwise increasing production stack and the stepwise decreasing demand stack. Inter-temporal coupling due to ramping constraints is taken into account using a myopic mechanism that conditions the power available for each unit in one hour to the power output in the previous or the next hour. Once the 24 hours have been processed, minimum profit conditions imposed by the generating units are checked. If one or more of these constraints are violated, the unit with the highest violation is expelled from the market (not to be readmitted) and the whole procedure rerun. The clearing concludes once all units declaring minimum profit conditions meet these constraints.

The model in this section is formulated in order to achieve the same type of price equilibrium subject to the units not meeting profit constraints being expelled. There may be multiple equilibria of this type, and if this is the case, the solution of this model may or may not correspond to the solution of the Spanish market heuristic just described. From this point of view, these minimum profit conditions can be considered "outside" any of the generating company, the consumer, or the ISO optimization problems. This is the reason for adding them later, after the market equilibrium is presented, first as a mixed linear complementarity problem and then as an equivalent quadratic programming problem. Moreover, even if they could be included for example, as part of the generating company or the ISO problem, the optimality conditions for this problem would not apply due to the binary nature of these constraints. This is a more general issue which researchers have struggled with over the years, namely, solving a market equilibrium problem for which there are integer or more generally non-convex constraints. One must take great care to show that such an equilibrium actually makes sense in this setting. In dealing with problems with integer constraints, one approach adopted in O'Neill et al. [74] is to solve the related optimization problem to optimality, then add constraints forcing the integer variables to be at the optimal levels. The result is a linear (or convex) program which has a defendable interpretation for an equilibrium. That work however was more geared towards a centralized setting as opposed to a market as is the case with the equilibrium procedure presented in this section. We address these issues in more detail in Chapter 4.

3.3.2 Problem Formulation

The single-period equilibrium procedure including minimum profit conditions for generating units is formulated as follows.

First, generating companies, consumers and the ISO are modeled as solving the optimization problems presented in Subsection 3.2.1. Therefore, the KKT conditions of this set of optimization problems result in a mixed linear complementarity problem that corresponds to the one stated in Subsection 3.2.2. The solution to this mixed linear complementarity problem is obtained as an optimal solution of an equivalent quadratic programming problem [25, 59]. This equivalence is explained in Appendix A. Finally, we consider the minimum profit condition for each generating unit that declares such a condition and include them as constraints to the quadratic problem.

The minimum profit condition can be formulated as,

$$\sum_{b=1}^{N_{Gi}} \big(\rho_{n(i)} - \lambda^{\mathrm{C}}_{Gib}\big) P_{Gib} \geq K_i v_i;\ \forall i \in G^{\mathrm{M}}, \tag{3.60}$$

where K_i is a positive constant that represents minimum profit for generating unit i; and v_i is the on-line status for generating unit i. This equation is only imposed for the generating units that declare minimum profit conditions and whose set is indexed by G^{M}. Binary variable v_i should meet the following equations,

$$v_i \geq \frac{\sum_{b=1}^{N_{Gi}} P_{Gib}}{P_{Gi}^{\max}};\ \forall i \in G^{\mathrm{M}} \tag{3.61}$$

$$v_i \in 0, 1;\ \forall i \in G^{\mathrm{M}}. \tag{3.62}$$

Note that conditions (3.60)-(3.62) either enforce minimum profit ($v_i = 1$) or expel the generating unit from the market ($v_i = 0$).

3.3.2.1 Mixed Linear Complementarity Problem

The mixed linear complementarity problem defined by the optimality conditions for the problems of the generating companies, for the problems of the consumers and for the problem of the ISO that determine the market equilibrium is succinctly stated in compact form in Subsection 3.2.2, equations (3.18)-(3.33). The solution to this problem must also satisfy minimum profit conditions, equations (3.60)-(3.62), but these equations cannot be directly included as additional equations of the mixed linear complementarity problem because the structure of the problem would be altered as binary variables would be included in the problem, and consequently, it would no longer be a complementarity problem. Therefore, to achieve a solution that satisfies both the mixed linear complementarity problem and the minimum profit conditions, the complementarity problem is reformulated as a quadratic programming problem as it is shown in the following subsection.

3.3.2.2 Equivalent Quadratic Programming Problem

The solution of a mixed linear complementarity problem can also be found as a stationary point of a quadratic programming problem, see Appendix A. The mixed linear complementarity problem has a solution if and only if there is a global minimum of the quadratic problem with an objective function optimal value equal to zero.

This quadratic programming problem should be extended to include the minimum profit conditions, which are nonlinear constraints that include binary variables. This quadratic problem can be written as
Minimize

$$Z_{\text{QPP}} \tag{3.63}$$

subject to

$$0 \leq \lambda^{\text{C}}_{Gib} - \rho_{n(i)} + \alpha_i + \phi_{ib};\ \forall i \in G; b = 1, \ldots, N_{Gi} \tag{3.64}$$

$$0 \leq P^{\max}_{Gi} - \sum_{b=1}^{N_{Gi}} P_{Gib};\ \forall i \in G \tag{3.65}$$

$$0 \leq P^{\max}_{Gib} - P_{Gib};\ \forall i \in G; b = 1, \ldots, N_{Gi} - 1 \tag{3.66}$$

$$P_{Gib} \geq 0;\ \forall i \in G; b = 1, \ldots, N_{Gi} \tag{3.67}$$

$$\alpha_i \geq 0;\ \forall i \in G \tag{3.68}$$

$$\phi_{ib} \geq 0;\ \forall i \in G; b = 1, \ldots, N_{Gi} \tag{3.69}$$

$$0 \leq \rho_{n(j)} - \lambda^{\text{U}}_{Djk} - \sigma_j + \varphi_{jk};\ \forall j \in D; k = 1, \ldots, N_{Dj} \tag{3.70}$$

$$0 \leq \sum_{k=1}^{N_{Dj}} P_{Djk} - P^{\min}_{Dj};\ \forall j \in D \tag{3.71}$$

$$0 \leq P^{\max}_{Djk} - P_{Djk};\ \forall j \in D; k = 1, \ldots, N_{Dj} \tag{3.72}$$

$$P_{Djk} \geq 0;\ \forall j \in D; k = 1, \dots, N_{Dj} \tag{3.73}$$

$$\sigma_j \geq 0;\ \forall j \in D \tag{3.74}$$

$$\varphi_{jk} \geq 0;\ \forall j \in D; k = 1, \dots, N_{Dj} \tag{3.75}$$

$$0 = \lambda^{\mathrm{B}}_{Gib} - \rho_{n(i)} + \mu_{Gib};\ \forall i \in G; b = 1, \dots, N_{Gi} \tag{3.76}$$

$$0 = \rho_{n(j)} - \lambda^{\mathrm{B}}_{Djk} + \nu_{Djk};\ \forall j \in D; k = 1, \dots, N_{Dj} \tag{3.77}$$

$$0 \leq \rho_n\big[B_{nm} - \frac{1}{2}G_{nm}\Delta\delta(2l-1)\big] + B_{nm}\,\gamma_{nm} + \zeta^{+}_{nm,l};\ \forall n \in N; \\ \forall m \in \Omega_n; l = 1, \dots, L \tag{3.78}$$

$$0 \leq \rho_n\big[-B_{nm} - \frac{1}{2}G_{nm}\Delta\delta(2l-1)\big] - B_{nm}\,\gamma_{nm} + \zeta^{-}_{nm,l};\ \forall n \in N; \\ \forall m \in \Omega_n; l = 1, \dots, L \tag{3.79}$$

$$0 = -\sum_{i\in\theta_n}\sum_{b=1}^{N_{Gi}} \widetilde{P}_{Gib} + \sum_{j\in\vartheta_n}\sum_{k=1}^{N_{Dj}} \widetilde{P}_{Djk} + \sum_{m\in\Omega_n} B_{nm}\sum_{l=1}^{L}\big(\delta^{+}_{nm,l} - \delta^{-}_{nm,l}\big) \\ - \frac{1}{2}\sum_{m\in\Omega_n}\Big[G_{nm}\Delta\delta\sum_{l=1}^{L}(2l-1)\big(\delta^{+}_{nm,l} + \delta^{-}_{nm,l}\big)\Big];\ \forall n \in N \tag{3.80}$$

$$0 \leq P^{\max}_{nm} - B_{nm}\sum_{l=1}^{L}\big(\delta^{+}_{nm,l} - \delta^{-}_{nm,l}\big);\ \forall n \in N; \forall m \in \Omega_n \tag{3.81}$$

$$0 = P_{Gib} - \widetilde{P}_{Gib};\ \forall i \in G; b = 1, \dots, N_{Gi} \tag{3.82}$$

$$0 = P_{Djk} - \widetilde{P}_{Djk};\ \forall j \in D; k = 1, \dots, N_{Dj} \tag{3.83}$$

$$0 \leq \Delta\delta - \delta^{+}_{nm,l};\ \forall n \in N; \forall m \in \Omega_n; l = 1, \dots, L \tag{3.84}$$

$$0 \leq \Delta\delta - \delta^{-}_{nm,l};\ \forall n \in N; \forall m \in \Omega_n; l = 1, \dots, L \tag{3.85}$$

$$\delta^{+}_{nm,l} \geq 0;\ \forall n \in N; \forall m \in \Omega_n; l = 1, \dots, L \tag{3.86}$$

$$\delta^{-}_{nm,l} \geq 0;\ \forall n \in N; \forall m \in \Omega_n; l = 1, \dots, L \tag{3.87}$$

$$\gamma_{nm} \geq 0;\ \forall n \in N; \forall m \in \Omega_n \tag{3.88}$$

$$\zeta^{+}_{nm,l} \geq 0;\ \forall n \in N; \forall m \in \Omega_n; l = 1, \dots, L \tag{3.89}$$

$$\zeta^{-}_{nm,l} \geq 0;\ \forall n \in N; \forall m \in \Omega_n; l = 1, \dots, L \tag{3.90}$$

$$\sum_{b=1}^{N_{Gi}}\big(\rho_{n(i)} - \lambda^{\mathrm{C}}_{Gib}\big)P_{Gib} \geq K_i v_i;\ \forall i \in G^{\mathrm{M}} \tag{3.91}$$

$$v_i \geq \frac{\sum_{b=1}^{N_{Gi}} P_{Gib}}{P^{\max}_{Gi}};\ \forall i \in G^{\mathrm{M}} \tag{3.92}$$

$$v_i \in \{0, 1\};\ \forall i \in G^{\mathrm{M}}, \tag{3.93}$$

where Z_{QPP} corresponds to the sum of all the products of the inequality equations of the mixed linear complementarity problem, equations (3.18)-(3.33), and their respective dual variables, that is, the sum of the complementarity conditions. Notice that the objective function value, Z_{QPP}, is always bounded below by zero. The constraints of the quadratic programming problem are all the inequalities and equalities of the associated mixed linear complementarity problem without considering the complementarity conditions, and the minimum profit conditions.

It should be noted that equation (3.91) is nonlinear because its left-hand side is the sum of bilinear terms. Three different computational techniques are presented in Section 3.3.4 in order to attain a solution of this mixed-integer nonlinear programming problem.

3.3.3 Uniqueness, Multiple Dual Solutions and Infeasibility

Including minimum profit constraints may render a market equilibrium problem infeasible unless this problem has multiple dual solutions. Multiple dual solution case and infeasibility are treated below in Subsections 3.3.3.1 and 3.3.3.2, respectively.

3.3.3.1 Multiple Dual Solutions Case

Equilibrium problem (3.18)-(3.33) might have multiple dual solutions, that is, might have multiple prices. This case is illustrated in Figure 3.4(a). Note that there is a range of prices at which the power supplied is equal to the power demanded. In this situation, minimum profit-constrained equilibrium problem (3.63)-(3.93) generally results in a feasible problem, whose optimal solution meets minimum profit conditions.

3.3.3.2 Infeasible Case

Equilibrium problem (3.18)-(3.33) has a unique solution as regards prices, generations, demands and flows. By adding minimum profit constraints to this problem, we simply create infeasibilities, assuming that the minimum profit condition is not attained for this unique solution beforehand. However, it should be noted that for practical applications where market agents behave in a reasonable manner, these infeasibilities are generally negligible for electricity markets with market agents behaving in a reasonable manner. The reason for this is as follows. Power supply curves tend to be "hockey-stick" shaped around the market-clearing price in most practical markets, while demand curves are rather inelastic around the market-clearing price. The supply curve has a hockey-stick shape because many bids are made at zero price to ensure acceptance. The demand curve is rather inelastic due to the nature of electricity consumption. Moreover, the number of steps in the supply curve is usually large (72 units times 25 steps each results in

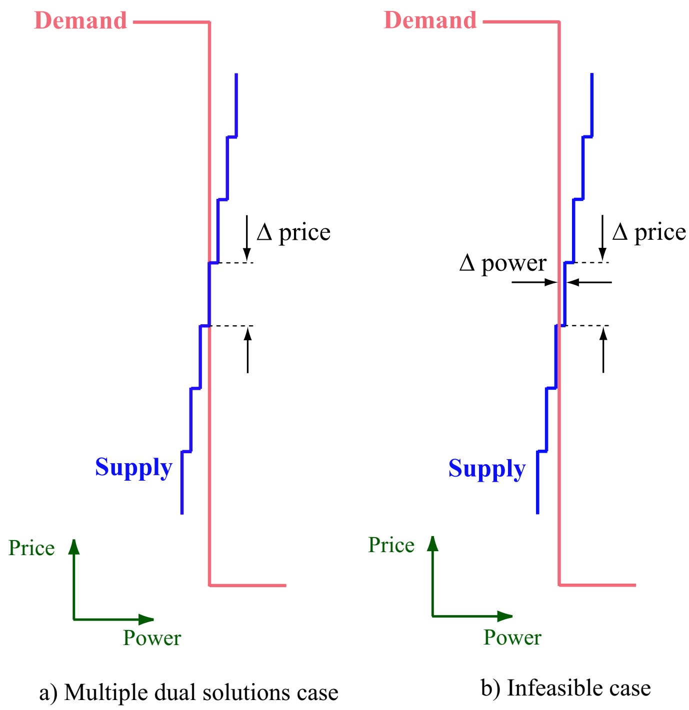

Figure 3.4: Multiple dual solutions and infeasible cases

1800 steps for the day-ahead market of mainland Spain). The above can be observed in the demand and supply curves shown in Figure 3.5 that correspond to the electricity market of mainland Spain (June 15, 2005, hour 11). The conclusion is that market-clearing is often in the vicinity of these "near-degeneracy" regions, in which there is a unique equilibrium albeit with step supply and demand curves as illustrated in Figure 3.4(b). In this figure, it can be observed that small increments in power (which create slight infeasibilities) result in significant price differences (which allow minimum profit conditions to be met).

Slight infeasibilities cause an optimal objective function value slightly different from zero in problem (3.63)-(3.93). These slight infeasibilities are related to prices because power balance is enforced at every node. The cost incurred due to price infeasibilities could be allocated pro-rata among market participants, (see [41]).

We define a near-equilibrium as an optimal solution of problem (3.63)-(3.93) that results in an optimal objective function value slightly different from zero, which implies that one or more of the complementarity conditions of the equilibrium problem (3.18)-(3.33) are slightly not satisfied.

Thus, in general, an optimal solution of problem (3.63)-(3.93) represents a near-equilibrium that may include small complementary infeasibilities of negligible practical significance. An appropriate measure of the importance

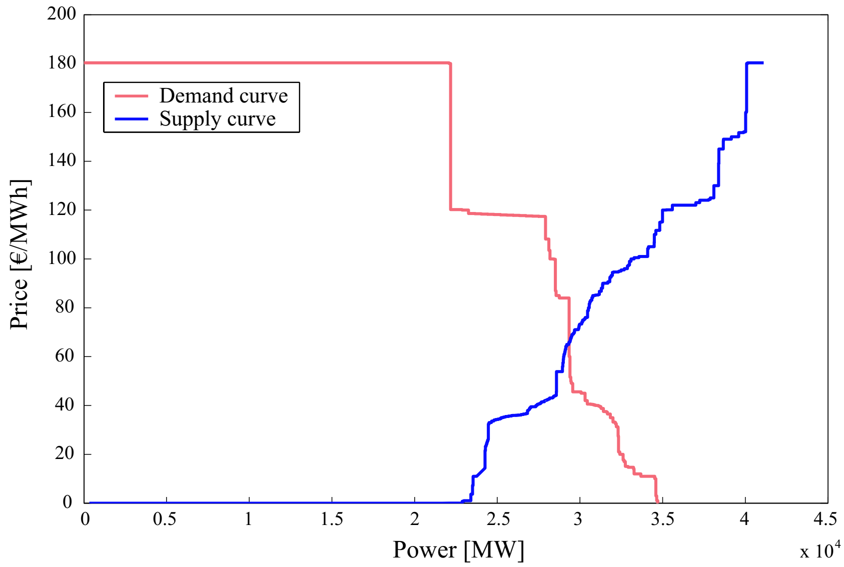

Figure 3.5: Demand and supply curves in the electricity market of mainland Spain

of such infeasibilities is the optimal value of the objective function (3.63). The closer to zero the optimal value of (3.63) is, the closer it is to the complementary feasibility problem (3.63)-(3.93).

3.3.3.3 Infeasibility Cost

As stated above, we can create slight infeasibilities in the equilibrium by adding minimum profit conditions. Some of these infeasibilities can be originated by the fact that a generating unit is paid at a price lower than its cost, or that a demand pays a price higher than its utility, provoking an economic loss in the generating unit or demand, respectively. The set of this kind of economic loss for every generating unit and demand of the market is called the infeasibility cost of the problem, which is usually insignificant.

We propose to cover such infeasibility cost (not covered by prices) paying an uplift to the generating units and demands with losses that are equal to its corresponding economic loss. Then, the total cost of these uplifts, C, could be assigned half to generating units and half to demands, and each half cost is allocated pro-rata among generating units or among demands. Other more elaborate procedures are also possible.

Therefore, the additional cost of each generating unit, C_i, and of each demand, C_j, are computed using the following expressions,

$$C_i = \frac{C}{2} \frac{\sum_{b=1}^{N_{Gi}} P_{Gib}}{\sum_{i=1}^{N_G} \sum_{b=1}^{N_{Gi}} P_{Gib}}; \; \forall i \in G; \forall i \notin G^{\text{uplift}} \tag{3.94}$$

$$C_i = \frac{C}{2} \frac{\sum_{b=1}^{N_{Gi}} P_{Gib}}{\sum_{i=1}^{N_G} \sum_{b=1}^{N_{Gi}} P_{Gib}} - \text{Uplift}_i; \; \forall i \in G^{\text{uplift}} \tag{3.95}$$

$$C_j = \frac{C}{2} \frac{\sum_{k=1}^{N_{Dj}} P_{Djk}}{\sum_{j=1}^{N_D} \sum_{k=1}^{N_{Dj}} P_{Djk}}; \; \forall j \in D; \forall i \notin D^{\text{uplift}} \tag{3.96}$$

$$C_j = \frac{C}{2} \frac{\sum_{k=1}^{N_{Dj}} P_{Djk}}{\sum_{j=1}^{N_D} \sum_{k=1}^{N_{Dj}} P_{Djk}} - \text{Uplift}_j; \; \forall i \in D^{\text{uplift}}, \tag{3.97}$$

where G^{uplift} and D^{uplift} represent the set of generating units and demands, respectively, that are compensated with an uplift; and Uplift_i and Uplift_j are the uplift paid to generating unit i and demand j, respectively.

Note that additional cost, C_i, is subtracted from the corresponding profit of generating unit i, and cost, C_j is added to the corresponding demand cost of demand j.

3.3.4 Solution Technique

Single-period near-equilibrium is obtained through the solution of the problem (3.63)-(3.93). This problem is a nonlinear problem which is difficult to solve. The main difficulty lies in the nonlinearity of the minimum profit constraints (3.91). We suggest three possible alternative procedures in order to solve this nonlinear problem.

a) To directly solve the problem using an appropriate nonlinear solver.

b) To linearize the nonlinear minimum profit constraints using Schur's decomposition and binary variables as stated in [38, 41], and to solve the resulting mixed-integer quadratic problem.

c) To use an algorithm based on a successive over-relaxation method [42].

When the dimension of the single-period equilibrium problem is large, the CPU time to directly solve the problem is very long, while the other two procedures work better. Next, we describe the procedure based on a linearization of the minimum profit constraints and the procedure based on a successive-over relaxation method.

3.3.4.1 Solution Technique: Linear Approximation of Minimum Profit Conditions

In this subsection, we introduce a new approach for handling particular types of non-convex functions but with general importance, such as bilinear functions [38]. We approximate these bilinear functions using only linear equations and binary variables that satisfy particular conditions (known as SOS type 2 variables).

This technique linearizes the minimum profit conditions declared for the generating units using Schur's decomposition [53] and binary variables [90], turning the problem (3.63)-(3.93) into a mixed-integer quadratic programming problem, which can be solved using appropriate solvers, e.g. SBB [40] or CPLEX [40, 57].

The linearization of the minimum profit conditions is outlined below:

a) The nonseparable quadratic terms (also called bilinear terms) of the minimum profit conditions are transformed into the sum of separable quadratic forms. This transformation makes use of Schur's theorem [53].

b) These separable quadratic terms can be approximated by piecewise linear functions with appropriate integer constraints [90].

The linear approximation of the minimum profit conditions is explained below.

Consider the minimum profit condition,

$$\sum_{b=1}^{N_{Gi}} \left(\rho_{n(i)} - \lambda_{Gib}^{\mathrm{C}}\right) P_{Gib} \geq K_i v_i; \ \forall i \in G^{\mathrm{M}}. \tag{3.98}$$

The left-hand side of (3.98) is the sum of linear terms, $\sum_{b=1}^{N_{Gi}}(-\lambda_{Gib}^{\mathrm{C}})P_{Gib}$, and bilinear terms, $\sum_{b=1}^{N_{Gi}} \rho_{n(i)} P_{Gib}$. The bilinear terms can be expressed more compactly as $\frac{1}{2}\boldsymbol{r}_i^{\mathrm{T}} H \boldsymbol{r}_i$, where $\boldsymbol{r}_i^{\mathrm{T}} = (\rho_{n(i)} \ \ P_{Gi1} \ \ \ldots \ \ P_{GiN_{Gi}})$ and where H is the Hessian matrix of the sum of bilinear terms and has the form $H = \begin{pmatrix} 0 & \boldsymbol{e}^{\mathrm{T}} \\ \boldsymbol{e} & 0 \end{pmatrix}$, where $\boldsymbol{e}^{\mathrm{T}} = (1 \ \ \ldots \ \ 1)$. Note that matrix $H \in \mathbb{R}^{(N_{Gi}+1)\times(N_{Gi}+1)}$ and vector $\boldsymbol{e}^{\mathrm{T}} \in \mathbb{R}^{(N_{Gi}+1)}$.

The following lemma describes Schur's decomposition that involves the determination of the eigenvalues of the Hessian matrix of the bilinear terms. Schur's decomposition of this matrix is a well-known result in linear algebra. An advantage of this decomposition is that it involves orthogonal matrices which are known to have good numerical stability properties.

Lemma 3.1 (Schur's decomposition). Let H be an $(n+1) \times (n+1)$ matrix of the form $H = \begin{pmatrix} 0 & \boldsymbol{e}^{\mathrm{T}} \\ \boldsymbol{e} & 0 \end{pmatrix}$, where $\boldsymbol{e}^{\mathrm{T}} \in \mathbb{R}^n$ is the vector of all ones.

Then,

- The eigenvalues of H are

  $\{\lambda_1, \ldots, \lambda_{n-1}, \lambda_n, \lambda_{n+1}\} = \{0, \ldots, 0, \sqrt{n}, -\sqrt{n}\}.$

- $H = QDQ^{\mathrm{T}}$ where $D = \mathrm{diag}(\lambda_1, \ldots, \lambda_{n-1}, \lambda_n, \lambda_{n+1})$ and Q is the orthogonal matrix given by

$$Q = \begin{pmatrix}
0 & 0 & & 0 & & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & & \frac{1}{\sqrt{j+j^2}} & & \frac{1}{\sqrt{(n-1)+(n-1)^2}} & \frac{1}{\sqrt{2n}} & \frac{1}{\sqrt{2n}} \\
-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & & \frac{1}{\sqrt{j+j^2}} & & \frac{1}{\sqrt{(n-1)+(n-1)^2}} & \frac{1}{\sqrt{2n}} & \frac{1}{\sqrt{2n}} \\
0 & -\frac{2}{\sqrt{6}} & \cdots & \vdots & \cdots & \frac{1}{\sqrt{(n-1)+(n-1)^2}} & \frac{1}{\sqrt{2n}} & \frac{1}{\sqrt{2n}} \\
0 & 0 & & \frac{1}{\sqrt{j+j^2}} & & \frac{1}{\sqrt{(n-1)+(n-1)^2}} & \frac{1}{\sqrt{2n}} & \frac{1}{\sqrt{2n}} \\
0 & 0 & & \frac{-j}{\sqrt{j+j^2}} & & \frac{1}{\sqrt{(n-1)+(n-1)^2}} & \frac{1}{\sqrt{2n}} & \frac{1}{\sqrt{2n}} \\
0 & 0 & & 0 & & \vdots & \frac{1}{\sqrt{2n}} & \frac{1}{\sqrt{2n}} \\
\vdots & \vdots & \cdots & \vdots & & \frac{1}{\sqrt{(n-1)+(n-1)^2}} & \vdots & \vdots \\
0 & 0 & & 0 & & \frac{-(n-1)}{\sqrt{(n-1)+(n-1)^2}} & \frac{1}{\sqrt{2n}} & \frac{1}{\sqrt{2n}}
\end{pmatrix}. \tag{3.99}$$

■

A proof of this lemma is reported in [38].

Schur's decomposition is applied to the bilinear terms $\frac{1}{2}\boldsymbol{r}_i^{\mathrm{T}} H \boldsymbol{r}_i$. Since H is a real symmetric matrix, Schur's decomposition states that there is an orthogonal matrix Q such that $H = QDQ^{\mathrm{T}}$, where D is a diagonal matrix whose eigenvalues match those of H. It can be shown that these eigenvalues are $\{0, \ldots, 0, \sqrt{N_{Gi}}, -\sqrt{N_{Gi}}\}$ so that,

$$\frac{1}{2}\boldsymbol{r}_i^{\mathrm{T}} H \boldsymbol{r}_i = \frac{1}{2}\boldsymbol{r}_i^{\mathrm{T}} QDQ^{\mathrm{T}} \boldsymbol{r}_i = \frac{1}{2}\boldsymbol{x}_i^{\mathrm{T}} D \boldsymbol{x}_i = \\ \frac{1}{2}\sqrt{N_{Gi}}(x_{i,N_{Gi}-1})^2 - \frac{1}{2}\sqrt{N_{Gi}}(x_{iN_{Gi}})^2, \tag{3.100}$$

with the linear constraint $Q^{\mathrm{T}}\boldsymbol{r}_i = \boldsymbol{x}_i$ where $\boldsymbol{x}_i^{\mathrm{T}} = (x_{i0} \;\; \ldots \;\; x_{iN_{Gi}})$ and,

$$Q = \left(\begin{array}{cccccccc}
0 & 0 & & 0 & & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & & \frac{1}{\sqrt{j+j^2}} & & \frac{1}{\sqrt{(N_{Gi}-1)+(N_{Gi}-1)^2}} & \frac{1}{\sqrt{2N_{Gi}}} & \frac{1}{\sqrt{2N_{Gi}}} \\
-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & & \frac{1}{\sqrt{j+j^2}} & & \frac{1}{\sqrt{(N_{Gi}-1)+(N_{Gi}-1)^2}} & \frac{1}{\sqrt{2N_{Gi}}} & \frac{1}{\sqrt{2N_{Gi}}} \\
0 & -\frac{2}{\sqrt{6}} & \cdots & \vdots & \cdots & \frac{1}{\sqrt{(N_{Gi}-1)+(N_{Gi}-1)^2}} & \frac{1}{\sqrt{2N_{Gi}}} & \frac{1}{\sqrt{2N_{Gi}}} \\
0 & 0 & & \frac{1}{\sqrt{j+j^2}} & & \frac{1}{\sqrt{(N_{Gi}-1)+(N_{Gi}-1)^2}} & \frac{1}{\sqrt{2N_{Gi}}} & \frac{1}{\sqrt{2N_{Gi}}} \\
0 & 0 & & \frac{-j}{\sqrt{j+j^2}} & & \frac{1}{\sqrt{(N_{Gi}-1)+(N_{Gi}-1)^2}} & \frac{1}{\sqrt{2N_{Gi}}} & \frac{1}{\sqrt{2N_{Gi}}} \\
0 & 0 & & 0 & & \vdots & \frac{1}{\sqrt{2N_{Gi}}} & \frac{1}{\sqrt{2N_{Gi}}} \\
\vdots & \vdots & \cdots & \vdots & & \frac{1}{\sqrt{(N_{Gi}-1)+(N_{Gi}-1)^2}} & \vdots & \vdots \\
0 & 0 & & 0 & & \frac{-(n-1)}{\sqrt{(N_{Gi}-1)+(N_{Gi}-1)^2}} & \frac{1}{\sqrt{2N_{Gi}}} & \frac{1}{\sqrt{2N_{Gi}}}
\end{array} \right), \tag{3.101}$$

where j represents the column number of Q.

Consequently, (3.98) is equivalent to the following expression,

$$\frac{1}{2}\sqrt{N_{Gi}}\left[(x_{i,N_{Gi}-1})^2 - (x_{iN_{Gi}})^2\right] - \sum_{b=1}^{N_{Gi}} \lambda_{Gib}^{\mathrm{C}} P_{Gib} \geq K_i v_i; \quad \forall i \in G^{\mathrm{M}} \tag{3.102}$$

with the additional linear constraints,

$$Q^{\mathrm{T}}\boldsymbol{r}_i = \boldsymbol{x}_i; \; \forall i \in G^{\mathrm{M}}, \tag{3.103}$$

where Q is given by equation (3.101).

In the light of the fact that there are no constraints on variables, $(x_{i0}, \ldots, x_{iN_{Gi}-2})$, these additional linear constraints reduce to,

$$\frac{1}{\sqrt{2}}\rho_{n(i)} + \sum_{b=1}^{N_{Gi}} \frac{1}{\sqrt{2N_{Gi}}} P_{Gib} = x_{i,N_{Gi}-1}; \; \forall i \in G^{\mathrm{M}} \tag{3.104}$$

$$-\frac{1}{\sqrt{2}}\rho_{n(i)} + \sum_{b=1}^{N_{Gi}} \frac{1}{\sqrt{2N_{Gi}}} P_{Gib} = x_{iN_{Gi}}; \; \forall i \in G^{\mathrm{M}}. \tag{3.105}$$

Therefore, equation (3.98) is equivalent to quadratic equation (3.102), and linear equations (3.104) and (3.105).

Note that terms $\frac{1}{2}\sqrt{N_{Gi}}(x_{i,N_{Gi}-1})^2$ and $-\frac{1}{2}\sqrt{N_{Gi}}(x_{iN_{Gi}})^2$ can be approximated with binary variables as we explain after the illustrative example 3.1.

The following example illustrates how Schur's theorem can be used.

Illustrative Example 3.1 (Schur's decomposition). We suppose that the generating unit 2 of example 3.2.5, which is located at node 2, imposes a minimum profit condition, that can be formulated as,

$$\left(\rho_2 - \lambda_{G21}^{\mathrm{C}}\right)P_{G21} + \left(\rho_2 - \lambda_{G22}^{\mathrm{C}}\right)P_{G22} \geq K_2 v_2. \tag{3.106}$$

As $\boldsymbol{r}_2^{\mathrm{T}} = (\rho_2 \ \ P_{G21} \ \ P_{G22})$ and $H = \left(\begin{smallmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{smallmatrix}\right)$, the bilinear terms of the minimum profit condition can be expressed as,

$$\rho_2 P_{G21} + \rho_2 P_{G22} = \frac{1}{2}(\rho_2 \ \ P_{G21} \ \ P_{G22}) \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \rho_2 \\ P_{G21} \\ P_{G22} \end{pmatrix}. \tag{3.107}$$

Considering the following orthogonal and diagonal matrices,

$$Q = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{4}} & \frac{1}{\sqrt{4}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{4}} & \frac{1}{\sqrt{4}} \end{pmatrix} \quad \text{and} \quad D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & -\sqrt{2} \end{pmatrix},$$

bilinear terms (3.107) can be rewritten as,

$$\rho_2 P_{G21} + \rho_2 P_{G22} = \frac{1}{2}\sqrt{2}(x_{21})^2 - \frac{1}{2}\sqrt{2}(x_{22})^2, \tag{3.108}$$

with constraints

$$\begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{4}} & \frac{1}{\sqrt{4}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{4}} & \frac{1}{\sqrt{4}} \end{pmatrix} \begin{pmatrix} \rho_2 \\ P_{G21} \\ P_{G22} \end{pmatrix} = \begin{pmatrix} x_{20} \\ x_{21} \\ x_{22} \end{pmatrix}. \tag{3.109}$$

Note that equation (3.108) includes variables x_{21} and x_{22}, therefore only the last two constraints of (3.109) are needed, because variable x_{20} is not necessary to rewrite the minimum profit condition of generating unit 2. These constraints are the following,

$$\frac{1}{\sqrt{2}}\rho_2 + \frac{1}{\sqrt{4}}P_{G21} + \frac{1}{\sqrt{4}}P_{G22} = x_{21} \tag{3.110}$$

$$-\frac{1}{\sqrt{2}}\rho_2 + \frac{1}{\sqrt{4}}P_{G21} + \frac{1}{\sqrt{4}}P_{G22} = x_{22}. \tag{3.111}$$

To sum up, minimum profit condition (3.106) is equivalent to the quadratic equation (3.108), and linear equations (3.110)-(3.111).

■

The next step of this procedure is to convert terms $\frac{1}{2}\sqrt{N_{Gi}}(x_{i,N_{Gi}-1})^2$ and $-\frac{1}{2}\sqrt{N_{Gi}}(x_{iN_{Gi}})^2$ into piecewise linear ones. This is accomplished via binary variables as follows.

Consider the breakpoints (points where the slope of the piecewise linear function changes) $b_{i1}, b_{i2}, \ldots, b_{iP}$ for functions $\frac{1}{2}\sqrt{N_{Gi}}(x_{i,N_{Gi}-1})^2$ and $-\frac{1}{2}\sqrt{N_{Gi}}(x_{iN_{Gi}})^2$. Note that separate breakpoints could also be used for each function. Then, $x_{i,N_{Gi}-1}$ and $x_{iN_{Gi}}$ can be written as,

$$x_{i,N_{Gi}-1} = \sum_{p=1}^{P} u_{1ip}\, b_{ip};\ \forall i \in G^{\mathrm{M}} \tag{3.112}$$

$$x_{iN_{Gi}} = \sum_{p=1}^{P} u_{2ip}\, b_{ip};\ \forall i \in G^{\mathrm{M}}. \tag{3.113}$$

Therefore, the terms $\frac{1}{2}\sqrt{N_{Gi}}(x_{i,N_{Gi}-1})^2$, $-\frac{1}{2}\sqrt{N_{Gi}}(x_{iN_{Gi}})^2$ are replaced by

$$\frac{1}{2}\sqrt{N_{Gi}}(x_{i,N_{Gi}-1})^2 = \frac{1}{2}\sqrt{N_{Gi}}\sum_{p=1}^{P} u_{1ip}\, b_{ip}^2;\ \forall i \in G^{\mathrm{M}} \tag{3.114}$$

$$-\frac{1}{2}\sqrt{N_{Gi}}(x_{iN_{Gi}})^2 = -\frac{1}{2}\sqrt{N_{Gi}}\sum_{p=1}^{P} u_{2ip}\, b_{ip}^2;\ \forall i \in G^{\mathrm{M}}, \tag{3.115}$$

where

$$\sum_{p=1}^{P} u_{1ip} = 1;\ \forall i \in G^{\mathrm{M}} \tag{3.116}$$

$$\sum_{p=1}^{P} u_{2ip} = 1;\ \forall i \in G^{\mathrm{M}} \tag{3.117}$$

$$0 \le u_{1ip} \le 1;\ \forall i \in G^{\mathrm{M}}; \forall p = 1, \ldots, P \tag{3.118}$$

$$0 \le u_{2ip} \le 1;\ \forall i \in G^{\mathrm{M}}; \forall p = 1, \ldots, P. \tag{3.119}$$

Figure 3.6 graphically shows the function $\frac{1}{2}\sqrt{N_{Gi}}(x_{i,N_{Gi}-1})^2$ and the corresponding linear approximation with respect to variable value $x_{i,N_{Gi}-1}$. Note that the actual function and the approximation coincide at the breakpoints. We continue explaining the behavior of this linearization. When the value of the variable $x_{i,N_{Gi}-1}$ corresponds to the value of a breakpoint, e.g. b_{im}, the variable values $u_{1ip};\ \forall p \neq m$ are equal to zero and u_{1im} is equal to one. If the value of the variable $x_{i,N_{Gi}-1}$ is between two consecutive breakpoints, e.g. b_{im} and $b_{i,m+1}$, then the variable values $u_{1ip};\ \forall p \neq \{m, m-1\}$ are equal to zero, and u_{1im}, $u_{1i,m+1}$ are different from zero and the sum of both values must be equal to one (see equation (3.116)). This explanation can also be applied to $-\frac{1}{2}\sqrt{N_{Gi}}(x_{iN_{Gi}})^2$.

To ensure that the two variables u_{1ip} different to zero are adjacent, we must add the following adjacency assumption. The adjacency constraints for

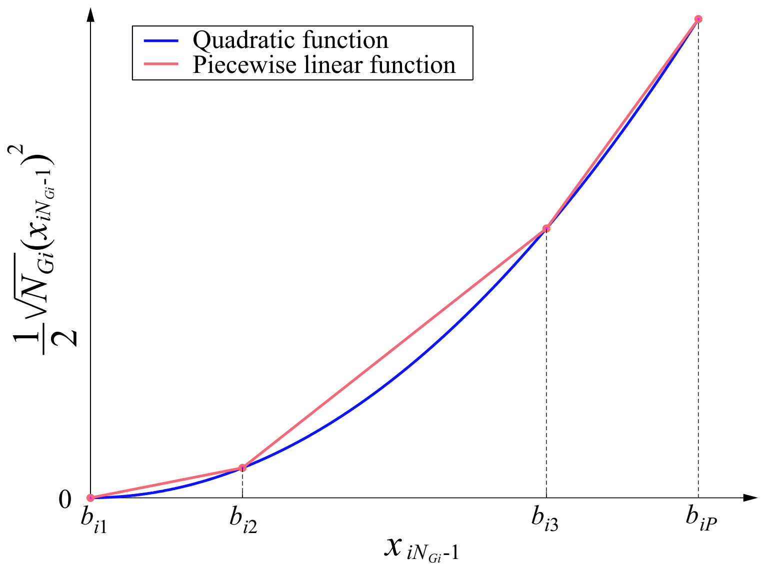

Figure 3.6: Piecewise linear approximation of a quadratic function

$x_{i,N_{Gi}-1}$ and $x_{iN_{Gi}}$ are as follows,

$$u_{1i1} \le z_{1i1}, u_{1i2} \le z_{1i1} + z_{1i2}, \ldots, u_{1i,P-1} \le z_{1i,P-2} + z_{1i,P-1}, \quad u_{1iP} \le z_{1i,P-1}; \ \forall i \in G^{\mathrm{M}} \tag{3.120}$$

$$u_{2i1} \le z_{2i1}, u_{2i2} \le z_{2i1} + z_{2i2}, \ldots, u_{2i,P-1} \le z_{2i,P-2} + z_{2i,P-1}, \quad u_{2iP} \le z_{2i,P-1}; \ \forall i \in G^{\mathrm{M}} \tag{3.121}$$

$$\sum_{p=1}^{P-1} z_{1ip} = 1; \ \forall i \in G^{\mathrm{M}} \tag{3.122}$$

$$\sum_{p=1}^{P-1} z_{2ip} = 1; \ \forall i \in G^{\mathrm{M}} \tag{3.123}$$

$$z_{1ip} \in \{0,1\}; \ \forall i \in G^{\mathrm{M}}; \forall p = 1, \ldots, P-1 \tag{3.124}$$

$$z_{2ip} \in \{0,1\}; \ \forall i \in G^{\mathrm{M}}; \forall p = 1, \ldots, P-1. \tag{3.125}$$

To see why this formulation works, observe that since $\sum_{p=1}^{P-1} z_{1ip} = 1$ and $z_{1ip} \in \{0,1\}$; $\forall p = 1, \ldots, P-1$, one of the z_{1ip}'s will be exactly equal to one, and the others will be equal to zero. And the adjacency constraint (3.120) implies that if $z_{1im} = 1$, then u_{1im} and $u_{1i,m+1}$ may be positive, but the other u_{1ip}'s must be equal to zero. These adjacent constraints ensure that if $b_{im} \le x_{i,N_{Gi}-1} \le b_{i,m+1}$, then $z_{1im} = 1$ and only u_{1im} and $u_{1i,m+1}$ can be positive.

To sum up, the following equations (3.126)-(3.138) are the linearization of the minimum profit condition (3.98):

$$\frac{1}{2}\sqrt{N_{Gi}}\left[\sum_{p=1}^{P} u_{1ip}(b_{ip})^2 - \sum_{p=1}^{P} u_{2ip}(b_{ip})^2\right] - \sum_{b=1}^{N_{Gi}} \lambda_{Gib}^{\mathrm{C}} P_{Gib} \geq K_i v_i;$$
$$\forall i \in G^{\mathrm{M}} \tag{3.126}$$

$$\frac{1}{\sqrt{2}}\rho_{n(i)} + \sum_{b=1}^{N_{Gi}} \frac{1}{\sqrt{2N_{Gi}}} P_{Gib} = \sum_{p=1}^{P} u_{1ip} b_{ip};\ \forall i \in G^{\mathrm{M}} \tag{3.127}$$

$$-\frac{1}{\sqrt{2}}\rho_{n(i)} + \sum_{b=1}^{N_{Gi}} \frac{1}{\sqrt{2N_{Gi}}} P_{Gib} = \sum_{p=1}^{P} u_{2ip} b_{ip};\ \forall i \in G^{\mathrm{M}} \tag{3.128}$$

$$\sum_{p=1}^{P} u_{1ip} = 1;\ \forall i \in G^{\mathrm{M}} \tag{3.129}$$

$$\sum_{p=1}^{P} u_{2ip} = 1;\ \forall i \in G^{\mathrm{M}} \tag{3.130}$$

$$0 \leq u_{1ip} \leq 1;\ \forall i \in G^{\mathrm{M}}; \forall p = 1, \ldots, P \tag{3.131}$$

$$0 \leq u_{2ip} \leq 1;\ \forall i \in G^{\mathrm{M}}; \forall p = 1, \ldots, P \tag{3.132}$$

$$u_{1i1} \leq z_{1i1}, u_{1i2} \leq z_{1i1} + z_{1i2}, \ldots, u_{1i,P-1} \leq z_{1i,P-2} + z_{1i,P-1},$$
$$u_{1iP} \leq z_{1i,P-1};\ \forall i \in G^{\mathrm{M}} \tag{3.133}$$

$$u_{2i1} \leq z_{2i1}, u_{2i2} \leq z_{2i1} + z_{2i2}, \ldots, u_{2i,P-1} \leq z_{2i,P-2} + z_{2i,P-1},$$
$$u_{2iP} \leq z_{2i,P-1};\ \forall i \in G^{\mathrm{M}} \tag{3.134}$$

$$\sum_{p=1}^{P-1} z_{1ip} = 1;\ \forall i \in G^{\mathrm{M}} \tag{3.135}$$

$$\sum_{p=1}^{P-1} z_{2ip} = 1;\ \forall i \in G^{\mathrm{M}} \tag{3.136}$$

$$z_{1ip} \in \{0, 1\};\ \forall i \in G^{\mathrm{M}}; \forall p = 1, \ldots, P-1 \tag{3.137}$$

$$z_{2ip} \in \{0, 1\};\ \forall i \in G^{\mathrm{M}}; \forall p = 1, \ldots, P-1. \tag{3.138}$$

The solution of the mixed-integer quadratic programming problem (3.63)-(3.93), replacing constraint (3.91) by equations (3.126)-(3.138) provides the market near-equilibrium under minimum profit conditions where profit for each generating unit, utility for each demand and social welfare are jointly maximized.

Numerical simulations using different electricity systems show the existence of the solution; however, no formal proof of its existence has been developed.

3.3.4.2 Solution Technique: Successive Over-Relaxation Iterative Method

There are two approaches that can be applied to solve linear systems of equations, namely

- Direct Methods.
- Iterative Methods.

Direct methods attempt to obtain the solution of a system of equations by performing a finite number of operations to arrive at the solution.

Iterative methods begin with an approximate solution, and incorporate the initial approximation in a recursive formula which generates another approximate solution. Under suitable conditions this sequence of solutions should converge to the exact solution.

The successive over-relaxation (SOR) method is an iterative method for solving a linear system of equations $Ax = b$, derived by extrapolating the Gauss-Seidel method. This extrapolation takes the form of a weighted average between the previous iterate and the computed Gauss-Seidel iterate successively for each component,

$$x_i^\nu = w\bar{x}_i^\nu + (1 - w)x_i^{\nu-1}, \tag{3.139}$$

where $\bar{x}_i^\nu$ denotes a Gauss-Seidel iterate and w is the extrapolation factor. The idea is to choose a value for w that accelerates the rate of convergence to the solution.

In general, it is not possible to compute the value of w that maximizes the rate of convergence of SOR in advance. Frequently, some heuristic estimate is used. From an experimental point of view, this successive over-relaxation algorithm presents good convergence behavior. A characterization of its convergence characteristic can be constructed based on results reported in [23, 45, 72, 79].

The equilibrium problem including minimum profit conditions is a mixed-integer nonlinear programming problem which is difficult to solve directly. The main difficulty lies in the nonlinearity of the minimum profit conditions. In this subsection, the iterative method proposed to solve the near-equilibrium is based on the idea of the successive over-relaxation iterative method. The description of this iterative method is as follows:

a) First, the equilibrium problem without considering minimum profit conditions is solved. The solution to this problem is used to compute an initial estimate of the generating powers.

b) The generating power values appearing in the minimum profit conditions are fixed to given values. This fact means that the minimum profit conditions turn into linear equations and therefore, the equilibrium problem turns into a mixed-integer quadratic programming problem.

c) The equilibrium problem formulated in b) is solved and relevant generating power values are updated.

d) Points b) and c) are repeated iteratively until the difference between the values of the generating power in two consecutive iterations is small enough.

This iterative method is formally described below.

Algorithm 3.1 (Successive over-relaxation iterative method).

Step 0: Initialization. Initialize the iteration counter, $\eta = 1$. Solve the equilibrium problem without considering minimum profit conditions, as defined by equations (3.63)-(3.90). If the solution to this problem satisfied all the minimum profit requirements imposed by all generating units, the solution to the single-period equilibrium has been attained and the successive over-relaxation method concludes. Otherwise (any minimum profit requirement is violated), the method continues.

The solution of this problem is used to compute an initial estimate of the generating powers. That is,

$$\hat{P}_{Gib}^{(1)} = a\bar{P}_{Gib}^{(1)};\ \forall i \in G^{\mathrm{M}}, \tag{3.140}$$

where $\hat{P}_{Gib}^{(1)}$ represents the initial estimate of the generating power of block b of unit i; $\bar{P}_{Gib}^{(1)}$ is the optimal generating power value of block b of generating unit i for the problem (3.63)-(3.90); and $a \geq 1$ is a constant.

Step 1: Equilibrium including minimum profit conditions. The generating power values appearing in the minimum profit conditions are fixed to the corresponding estimated values. Therefore, the minimum profit conditions (3.91) turn into linear expressions and the single-period equilibrium problem (3.63)-(3.93) becomes a mixed-integer quadratic program which is solved. The minimum profit conditions considered in the single-period equilibrium problem take the following form

$$\sum_{b=1}^{N_{Gi}} \left(\rho_{n(i)} - \lambda_{Gib}^{\mathrm{C}}\right)\hat{P}_{Gib}^{(\eta)} \geq K_i v_i;\ \forall i \in G^{\mathrm{M}}. \tag{3.141}$$

Note that equation (3.141) is a linear equation because the values of the generating powers are fixed to given values. This equation and expressions (3.92) and (3.93) are included as constraints to the quadratic problem presented in Step 0. The optimal generating power values for this problem are $\bar{P}_{Gib}^{(\eta+1)}$.

Step 2: Generating power estimate updating. Update the estimates of the generating powers through equation

$$\hat{P}_{Gib}^{(\eta+1)} = d\bar{P}_{Gib}^{(\eta+1)} + (1-d)\hat{P}_{Gib}^{(\eta)}; \ \forall i \in G^{\mathrm{M}}, \tag{3.142}$$

where the constant $d \in (0,1)$. Note that constant d does not change with each iteration.

If for all $i \in G^{\mathrm{M}}$, $\left| \dfrac{\hat{P}_{Gib}^{(\eta+1)} - \hat{P}_{Gib}^{(\eta)}}{\hat{P}_{Gib}^{(\eta)}} \right| \leq \epsilon$, stop, the solution has been found and corresponds to the solution of Step 1. If this is not the case, the iteration counter is updated, $\eta \leftarrow \eta + 1$ and the algorithm continues in Step 1.

Note that ϵ is an appropriate convergence tolerance.

■

Numerical simulations using different power systems show the appropriate convergence behavior of the solution; however, no formal proof of convergence has been developed. Based on these simulations, we can state that constants a and d have influence in the speed of convergence of the method.

3.3.5 Problem Size

The size of the problem solved to obtain the single-period near-equilibrium if we use the linear approximation of the minimum profit constraints is illustrated in Table 3.9. In this table, $N_{G^{\mathrm{M}}}$ represents the total number of the units that impose a minimum profit requirement; and P represents the number of blocks used to linearize the minimum profit conditions.

Table 3.9: Size of the QPP with linearized minimum profit conditions

Number of continuous variables	Number of binary variables	Number of constraints
$4(N_{GB}+N_{DK})+2N_L$ $+N_G+N_D+N_N$ $+8N_LL+2PN_{G^{\mathrm{M}}}$	$2PN_{G^{\mathrm{M}}}-N_{G^{\mathrm{M}}}$	$6(N_{GB}+N_{DK})+N_N+4N_L$ $+2N_D+16N_LL$ $+8N_{G^{\mathrm{M}}}+4PN_{G^{\mathrm{M}}}-2N_{G^{\mathrm{M}}}$

The size of the problem solved in each iteration of the successive over-relaxation algorithm in order to compute the single-period near-equilibrium is illustrated in Table 3.10.

The QPP with linearized minimum profit conditions is larger than the QPP with fixed minimum profit conditions.

Table 3.10: Size of the QPP with fixed minimum profit conditions

Number of continuous variables	Number of binary variables	Number of constraints
$4(N_{GB}+N_{DK})$ $+2N_L+N_G+N_D$ $+N_N+8N_LL$	$N_{G^{\mathrm{M}}}$	$6(N_{GB}+N_{DK})+N_N$ $+4N_L+N_G+2N_D$ $+16N_LL+2N_{G^{\mathrm{M}}}$

Commercial solvers for solving mixed-integer quadratic programming problems include, among others, CPLEX [40, 57], SBB [40] and DICOPT [40], which can be used under GAMS [14] or AMPL [36].

3.4 Economic Efficiency Metrics

There are some metrics to evaluate the economic efficiency of the market equilibrium. These measures are mainly the producer surplus, the consumer surplus, social welfare and the merchandising surplus. These measures are defined below.

The producer surplus is used to measure the welfare of the group of generating companies selling electric power at a particular price. The producer surplus is defined as the difference between what the generating companies actually receive for selling the power and the minimum amount that they would have to receive in order to supply the given level of power output. Producer surplus can be computed using the following equation:

$$\mathrm{PS} = \sum_{i\in G}\sum_{b=1}^{N_{Gi}}\big(\rho_{n(i)} - \lambda^{\mathrm{C}}_{Gib}\big)P_{Gib}. \tag{3.143}$$

The consumer surplus is used to measure the welfare of the group of consumers purchasing electric power at a particular price. The consumer surplus is defined as the difference between what consumers are willing to pay for the power they buy and the amount that consumers actually pay. The following equation represents the consumer surplus:

$$\mathrm{CS} = \sum_{j\in D}\sum_{k=1}^{N_{Dj}}\big(\lambda^{\mathrm{U}}_{Djk} - \rho_{n(j)}\big)P_{Djk}. \tag{3.144}$$

The merchandising surplus is defined as the difference between the total demand costs and the revenues of all the generating units. This amount appears during congested periods and it is used to pay to the transmission provider. The merchandising surplus is expressed by the following equation:

$$\mathrm{MS} = \sum_{j \in D} \sum_{k=1}^{N_{Dj}} \rho_{n(j)} P_{Djk} - \sum_{i \in G} \sum_{b=1}^{N_{Gi}} \rho_{n(i)} P_{Gib}. \tag{3.145}$$

Finally, the declared social welfare is defined as the total profits to the buyers (the consumers) minus the total costs to the sellers (the generating companies), and is obtained through the expression below:

$$\mathrm{DSW} = \sum_{j \in D} \sum_{k=1}^{N_{Dj}} \lambda^{\mathrm{B}}_{Djk} P_{Djk} - \sum_{i \in G} \sum_{b=1}^{N_{Gi}} \lambda^{\mathrm{B}}_{Gib} P_{Gib}. \tag{3.146}$$

It should be noted that if the generating companies do not bid at their respective marginal costs, the second term of the objective function is not actually the cost of the generating unit. And if the consumers do not bid at their respective marginal utilities, the first term is not the profit of the consumer. Therefore, the above expression represents the declared social welfare. The actual social welfare is computed as the sum of the producer surplus, the consumer surplus and the merchandising surplus:

$$\mathrm{SW} = \mathrm{PS} + \mathrm{CS} + \mathrm{MS}. \tag{3.147}$$

3.5 Example

This example illustrates the solution procedures in Subsection 3.3.4 considering minimum profit conditions for the 4-node network presented in Subsection 3.2.5. The two solution techniques explained in Subsections 3.3.4.1 and 3.3.4.2 are used to solve the equilibrium problem.

3.5.1 Data

Topology, line, generating unit and demand data can be found in Subsection 3.2.5.1. Besides, generating unit 2 declares a minimum profit condition equal to 130 \$/h.

3.5.2 Linearization Algorithm

The equilibrium considering the minimum profit condition for the generating unit 2 is computed linearizing this minimum profit condition as stated in Subsection 3.3.4.1. The breakpoints of the linearization of the quadratic terms of the minimum profit condition are 12.4, 12.8, 13.2, 13.6, 14.0, 14.4, 14.8, 15.2, 15.6, 16.0 and 16.4.

Table 3.11 provides results for the near-equilibrium concerning generating unit power output, revenues and profits. This results are obtained solving the problem (3.63)-(3.93) after replacing constraint (3.91) by constraints (3.126)-(3.138). Note that the profit achieved by generating unit 2 is slightly higher than the imposed limit, which is 130 $/h.

Table 3.11: Results for the generating units. Linearization method for example 3.5

Unit	Power output [MW]	Revenue [$/h]	Profit [$/h]
1	100.00	2023.34	163.34
2	217.15	4353.84	130.78
Total	317.15	6377.18	294.12

Results of power consumed and demand costs of every demand are presented in Table 3.12.

Table 3.12: Results for the demands. Linearization method for example 3.5

Node	Power consumed [MW]	Demand cost [$/h]
3	214.14	4389.86
4	100.00	2032.19
Total	314.14	6422.05

Locational marginal prices for all nodes are shown in Table 3.13.

Table 3.13: Locational marginal prices. Linearization method for example 3.5

Node	Locational marginal price [$/MWh]
1	20.2334
2	20.0496
3	20.5000
4	20.3219

Finally, Table 3.14 presents results for lines.

Table 3.14: Results for lines. Linearization method for example 3.5

From node	To node	δ_{ij}^{+} [rad]	δ_{ij}^{-} [rad]	Power flow [MW]	Losses [MW]
1	3	0.0342	0.0000	65.28	0.4913
1	4	0.0133	0.0000	34.40	0.1501
2	3	0.0580	0.0000	150.00	1.7962
2	4	0.0436	0.0000	65.97	0.5757

The objective function value of the quadratic programming problem solved to obtain an optimal solution of the near-equilibrium is slightly different from zero (0.091 per unit), so small complementarity infeasibilities are present.

3.5.3 Successive Over-Relaxation Method

The equilibrium considering minimum profit conditions is also solved using the iterative method presented in Subsection 3.3.4.2. The value of constants a and d are set to 1.1 and 0.8, respectively. The relative tolerance imposed to stop the iterative method is 0.001. The near-equilibrium is achieved applying Algorithm 3.1. The iterative algorithm has reached the optimal solution in 5 iterations.

Results concerning generating units are provided in Table 3.15. Note that the profit for generating unit 2 is exactly 130 $/h, the minimum imposed, in contrast with the profit obtained with the linearization method, 130.78 $/h. The reason for this is that the minimum profit condition is satisfied in the iterative method, but the linearization technique satisfies an approximation of this condition.

Table 3.15: Results for the generating units. Iterative method for example 3.5

Unit	Power output [MW]	Revenue [$/h]	Profit [$/h]
1	100.00	2023.34	163.34
2	217.15	4353.04	130.00
Total	317.15	6376.38	293.32

Results concerning demand and lines are the same as the ones obtained with the linearization method, Tables 3.12 and 3.14. Locational marginal prices are provided in Table 3.16. The price in node 2 is lower than the price obtained for that node using the linearization method. This causes the profit of the generating unit 2 to be lower using the successive over-relaxation method than using the linearization one.

Table 3.16: Locational marginal prices. Iterative method for example 3.5

Node	Locational marginal price [$/MWh]
1	20.2334
2	20.0460
3	20.5000
4	20.3219

Finally, note that the minimum profit condition for generating unit 2 results in small complementarity infeasibilities since the objective function optimal value of the corresponding problem is 0.084 per unit.

3.5.4 Comparison with the no Minimum Profit Condition Case

Figure 3.7 shows revenues and profits for each generating unit for the cases with and without the minimum profit condition. The results represented in the figure considering the minimum profit condition are the ones obtained through the successive over-relaxation method. Note that there are changes in the profit of unit 2, but the rest of the results remain identical in both cases.

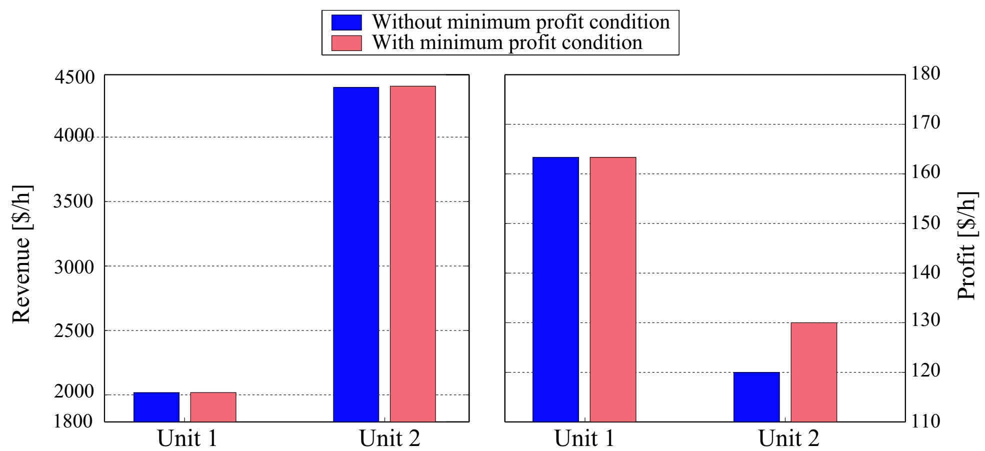

Figure 3.7: Comparison in terms of revenue and profit. Example 3.5

Figure 3.8 represents locational marginal prices throughout the network considering and not considering the minimum profit condition. Observe the changes in the locational marginal price of node 2 due to the minimum profit condition imposed by unit 2.

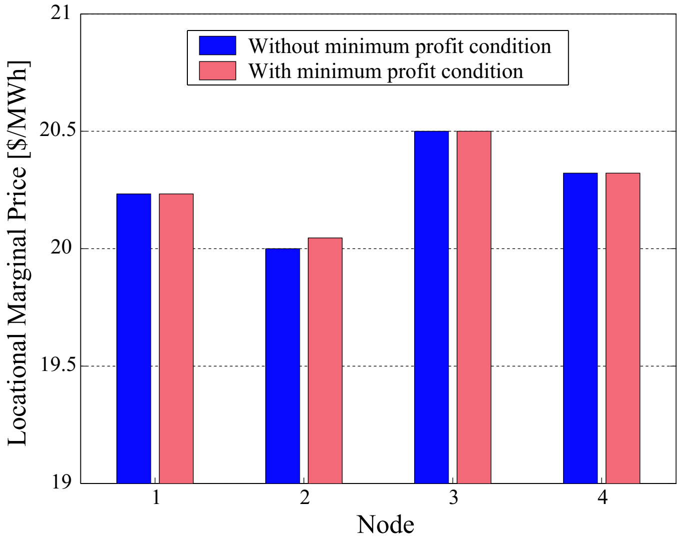

Figure 3.8: Comparison in terms of LMP. Example 3.5

The size of the problem solved if the linearization method is used to solve the single-period equilibrium is provided in the second row of the Table 3.17. The third row provides the size of the quadratic programming problem solved in each iteration of the successive over-relaxation method.

Table 3.17: Size of the QPP solved to obtain the single-period equilibrium for example 3.5

Method	Number of continuous variables	Number of binary variables	Number of constraints
Linearization	198	21	378
SOR	176	1	332

Finally, Table 3.18 provides an economic comparison between the single-period equilibrium with and without Minimum Profit Conditions (MPC) for the example. We observe that the consideration of minimum profit conditions implies an increase in the producer surplus and a decrease in the merchandising surplus. But both consumer surplus and social welfare do not change. Note that the declared social welfare coincides with the actual social welfare as the price bids of the generating companies and the consumers correspond to their marginal costs and utilities, respectively.

Table 3.18: Comparison of economic metrics. Example 3.5

	Single-period equilibrium without MPC	Single-period equilibrium with MPC	Difference [%]
Producer surplus [$]	283.34	293.32	3.5
Consumer surplus [$]	527.81	527.81	0.0
Merchandising surplus [$]	55.65	45.67	-17.9
Social welfare [$]	866.80	866.80	0.0

3.6 Summary

This chapter presents a model to obtain a single-period equilibrium for an electricity market. The single-period equilibrium is defined as the energy transaction levels and their associated prices that results in maximum profit for every individual generating company, maximum utility for every consumer, and maximum social welfare for the ISO. First, we do not consider minimum profit conditions in the equilibrium problem, and such equilibrium is described through the set of the KKT optimality conditions for the problems of all the generating companies, for the problems of all the consumers, and for the problem of the ISO, that result in a mixed linear complementarity problem. This market equilibrium is compared with results from an optimal power flow. Second, we include conditions in the equilibrium problem that ensure minimum profits for generating units. The mixed linear complementarity problem is formulated as an equivalent programming problem, and the minimum profit conditions are included as additional constraints turning the quadratic problem into a nonlinear problem. Three methodologies are developed to solve this nonlinear problem, specifically, a direct solution of the problem, a method that linearizes the minimum profit conditions, and a successive-over relaxation iterative method. We illustrate that these minimum profit conditions can cause that the equilibrium no longer exists; in such case, we compute a near-equilibrium.

Chapter 4

Multi-Period Equilibrium / Near-Equilibrium

4.1 Introduction

Based on the single-period equilibrium model developed in Chapter 3, a multi-period equilibrium model within a pool-based electricity market and a procedure to compute that multi-period equilibrium are proposed in this chapter.

As in Chapter 3, the pool-based electricity market includes generating companies, consumers and an Independent System Operator (ISO). The bidding stacks of every unit of each generating company and the bidding stacks of every demand of each consumer are submitted to the market operator, which clears the market maximizing the social welfare. The market-clearing procedure includes a representation of the network including losses and the effect of congestion, therefore the resulting prices are locational marginal prices [80].

A multi-period equilibrium is defined as the generating company / consumer energy transaction levels and their associated prices that result in maximum profit for every generating company, maximum utility for every consumer, and maximum social welfare for the whole multi-period framework, while inter-temporal constraints including the on / off status of the units and ramping limit constraints are enforced. This is illustrated in Figure 4.1. To avoid the limitations imposed by the necessary use of binary variables to model the on / off decisions, the equilibrium conditions are formulated through Benders decomposition [7, 21, 43], which allows the resulting equilibrium problem to be solved.

The multi-period equilibrium model allows the production schedules for the generating units to be obtained considering the operating, start-up and shut-down costs along the whole multi-period framework. These production schedules do not have to be the same as those obtained by finding the market equilibrium in every time period, without taking into account the rest of the

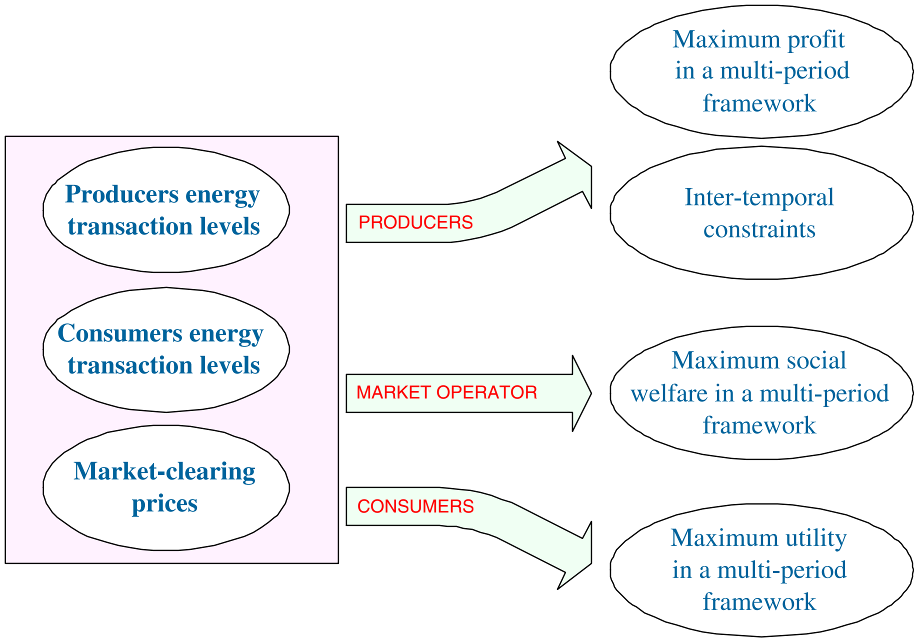

Figure 4.1: Definition of multi-period equilibrium

time periods. That is, the solution of a succession of single-period equilibrium problems can be different than the solution of one multi-period equilibrium problem as is showed by the results of Chapter 5, Subsection 5.3.2.

This chapter presents the formulation of a multi-period equilibrium problem in both cases, without considering and considering minimum profit conditions, and three procedures to solve each of these are proposed.

4.2 Limitations Imposed by Binary Decisions

As regards the single-period case, the equilibrium can be obtained considering the set of continuous optimization problems corresponding to the maximum profit of the generating companies, maximum utility of the consumers and the maximum social welfare of the independent system operator, and the corresponding optimality conditions [6] result in a mixed linear complementary problem [25] which is easy to solve [25, 50, 59].

On the contrary, for the multi-period case, the optimization problem corresponding to a generating company embodies binary decisions, i.e., on / off status for the units, which turn the problem into non-convex problem, and therefore optimality conditions are not sufficient for a global optimum [6]. To prevent such limitations while retaining the advantages of using optimality conditions, we define the multi-period equilibrium problem through Benders decomposition [7, 21, 43], which allows the separation of binary from continuous decisions and then, the computation of the multi-period equilibrium through the optimality conditions of the problems of the market agents while binary variables are fixed to given values. The binary variable values are improved iteratively until the optimum of the global problem is found. Therefore, each market agent maximizes its respective individual and conflicting objectives including both continuous and binary variables. The combination of complementarity theory and Benders decomposition is an advance in modeling. This procedure is explained in detail in the next section.

4.3 Equilibrium without Minimum Profit Constraints

The multi-period equilibrium results in the maximum profit for each generating company, the maximum utility for each consumer and the maximum social welfare throughout the multi-period framework satisfying inter-temporal constraints and including the binary variables that represent the on / off status for the units in the generating companies problems. Additionally, fixed, start-up and shut-down costs are considered, [3, 89, 91].

Roughly speaking, we can identify a market equilibrium as the outcome of a market economy in which each agent in the economy is doing as well as it can given the actions of the all other agents.

It should be noted that the model we present is equivalent in a centralized environment to a multi-period optimal power flow (see, for instance, [1, 5, 64]). Note also that the proposed formulation allows conditions involving dual variables to be imposed. This is not the case if a primal / dual approach such as Lagrangian relaxation is used.

This section formulates the multi-period equilibrium and describes the Benders decomposition algorithm used to find this equilibrium.

4.3.1 Formulation using Benders Decomposition

The multi-period equilibrium is defined by the optimization problems of each generating company, of each consumer and of the ISO. These optimization problems were formulated in Chapter 2. The equilibrium is obtained solving this set of problems simultaneously. But the KKT optimality conditions cannot be directly applied to the optimization problems because some of them include binary variables. Therefore, a decomposition technique is used. This technique is Benders decomposition [7, 21, 43], which fixes the binary variables to given values and allows using optimality conditions for the set of optimization problems to find a market equilibrium. A description of the Benders decomposition technique is provided in Appendix B.

If binary variables are fixed to given values, the multi-period equilibrium problem, corresponding to the status for the generating units defined by binary variables, can be solved through a Quadratic Programming Problem

(QPP), the subproblem. In turn, the binary variables are defined through a mixed-integer linear programming problem, the master problem.

The solution of the subproblem provides useful information about the "goodness" of the values of the binary variables related to the on / off status of the units, defined at the master problem level. In turn, this information is used by the master problem to refine the on / off status for the generating units of the generating companies. Benders decomposition is illustrated in Figure 4.2.

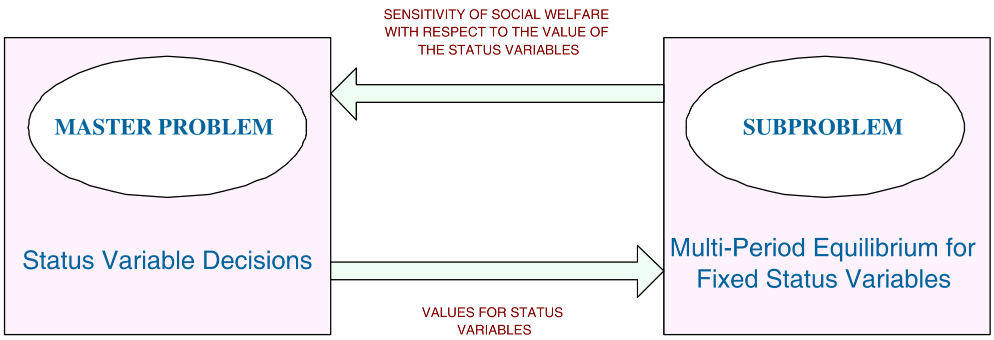

Figure 4.2: Structure of Benders decomposition

The formulations of the subproblem and the master problem are stated below.

4.3.1.1 Subproblem: Multi-Period Equilibrium for Fixed Status (Binary) Variables

If binary variables are fixed to given values, the multi-period equilibrium is determined by the Mixed Linear Complementarity Problem (MLCP) defined by the optimality conditions for the problems of the generating companies, the consumers and the ISO, conditions (2.55)-(2.64), (2.77)-(2.79) and (2.120)-(2.129), respectively. This mixed linear complementarity problem can be solved using an equivalent quadratic programming problem [25]. This is done considering all time periods. Moreover, to facilitate the decomposition and improve computational behavior, social welfare is subtracted from the objective function. Note that subtracting this term does not alter the solution to the problem, because through the problem of the ISO we already maximize the social welfare, but prevent a null objective function value that might cause convergence problems in the Benders master problem. The subproblem is formulated as follows,
Minimize

$$Z_{\mathrm{QPP}} - \sum_{t\in T}\sum_{j\in D}\sum_{k=1}^{N_{Dj}} \lambda^{\mathrm{B}}_{Djk}(t)P_{Djk}(t) + \sum_{t\in T}\sum_{i\in G}\sum_{b=1}^{N_{Gi}} \lambda^{\mathrm{B}}_{Gib}(t)P_{Gib}(t) \tag{4.1}$$

subject to

a) Optimality conditions of all problems of the generating companies:

$$0 \leq \lambda^{\mathrm{C}}_{Gib}(t) - \rho_{n(i)}(t) + \alpha_i(t) - \beta_i(t) + \phi_{ib}(t) + \tau_i(t) - \tau_i(t-1) + \psi_i(t-1) - \psi_i(t);\ \forall i \in G; b = 1, \ldots, N_{Gi}; \forall t \in T \tag{4.2}$$

$$0 \leq P^{\max}_{Gi}\overline{v}_i(t) - \sum_{b=1}^{N_{Gi}} P_{Gib}(t);\ \forall i \in G; \forall t \in T \tag{4.3}$$

$$0 \leq \sum_{b=1}^{N_{Gi}} P_{Gib}(t) - P^{\min}_{Gi}\overline{v}_i(t);\ \forall i \in G; \forall t \in T \tag{4.4}$$

$$0 \leq P^{\max}_{Gib}(t) - P_{Gib}(t);\ \forall i \in G; b = 1, \ldots, N_{Gi} - 1; \forall t \in T \tag{4.5}$$

$$0 \leq R^{\mathrm{up}}_i\overline{v}_i(t-1) + R^{\mathrm{su}}_i\big[\overline{v}_i(t) - \overline{v}_i(t-1)\big] + P^{\max}_{Gi}\big[1 - \overline{v}_i(t)\big] - \sum_{b=1}^{N_{Gi}} P_{Gib}(t) + \sum_{b=1}^{N_{Gi}} P_{Gib}(t-1);\ \forall i \in G; \forall t \in T \tag{4.6}$$

$$0 \leq R^{\mathrm{dn}}_i\overline{v}_i(t) + R^{\mathrm{sd}}_i\big[\overline{v}_i(t-1) - \overline{v}_i(t)\big] + P^{\max}_{Gi}\big[1 - \overline{v}_i(t-1)\big] - \sum_{b=1}^{N_{Gi}} P_{Gib}(t-1) + \sum_{b=1}^{N_{Gi}} P_{Gib}(t);\ \forall i \in G; \forall t \in T \tag{4.7}$$

$$P_{Gib}(t) \geq 0;\ \forall i \in G; b = 1, \ldots, N_{Gi}; \forall t \in T \tag{4.8}$$

$$\alpha_i(t) \geq 0;\ \forall i \in G; \forall t \in T \tag{4.9}$$

$$\beta_i(t) \geq 0;\ \forall i \in G; \forall t \in T \tag{4.10}$$

$$\phi_{ib}(t) \geq 0;\ \forall i \in G; b = 1, \ldots, N_{Gi}; \forall t \in T \tag{4.11}$$

$$\tau_i(t) \geq 0;\ \forall i \in G; \forall t \in T \tag{4.12}$$

$$\psi_i(t) \geq 0;\ \forall i \in G; \forall t \in T. \tag{4.13}$$

b) Optimality conditions of all problems of the consumers:

$$0 \leq \rho_{n(j)}(t) - \lambda^{\mathrm{U}}_{Djk}(t) - \sigma_j(t) + \varphi_{jk}(t); \forall j \in D; k = 1, \ldots, N_{Dj}; \forall t \in T \tag{4.14}$$

$$0 \leq \sum_{k=1}^{N_{Dj}} P_{Djk}(t) - P^{\min}_{Dj}(t);\ \forall j \in D; \forall t \in T \tag{4.15}$$

$$0 \leq P^{\max}_{Djk}(t) - P_{Djk}(t);\ \forall j \in D; k = 1, \ldots, N_{Dj}; \forall t \in T \tag{4.16}$$

$$P_{Djk}(t) \geq 0;\ \forall j \in D; k = 1, \ldots, N_{Dj}; \forall t \in T \tag{4.17}$$

$$\sigma_j(t) \geq 0;\ \forall j \in D; \forall t \in T \tag{4.18}$$

$$\varphi_{jk}(t) \geq 0;\ \forall j \in D; k = 1, \ldots, N_{Dj}; \forall t \in T. \tag{4.19}$$

c) Optimality conditions of the ISO problem:

$$0 = \lambda^{\mathrm{B}}_{Gib}(t) - \rho_{n(i)}(t) + \mu_{Gib}(t);\ \forall i \in G; b = 1, \dots, N_{Gi}; \forall t \in T \quad (4.20)$$

$$0 = \rho_{n(j)}(t) - \lambda^{\mathrm{B}}_{Djk}(t) + \nu_{Djk}(t);\ \forall j \in D; k = 1, \dots, N_{Dj}; \forall t \in T \quad (4.21)$$

$$0 \le \rho_n(t)\big[B_{nm} - \frac{1}{2}G_{nm}\Delta\delta(2l-1)\big] + B_{nm}\ \gamma_{nm}(t) + \zeta^{+}_{nm,l}(t); \forall n \in N; \forall m \in \Omega_n; l = 1, \dots, L; \forall t \in T \quad (4.22)$$

$$0 \le \rho_n(t)\big[-B_{nm} - \frac{1}{2}G_{nm}\Delta\delta(2l-1)\big] - B_{nm}\ \gamma_{nm}(t) + \zeta^{-}_{nm,l}(t); \forall n \in N; \forall m \in \Omega_n; l = 1, \dots, L; \forall t \in T \quad (4.23)$$

$$0 = -\sum_{i \in \theta_n}\sum_{b=1}^{N_{Gi}} \widetilde{P}_{Gib}(t) + \sum_{j \in \vartheta_n}\sum_{k=1}^{N_{Dj}} \widetilde{P}_{Djk}(t) + \sum_{m \in \Omega_n} B_{nm}\sum_{l=1}^{L}\big[\delta^{+}_{nm,l}(t) - \delta^{-}_{nm,l}(t)\big] - \frac{1}{2}\sum_{m \in \Omega_n}\Big[G_{nm}\Delta\delta\sum_{l=1}^{L}(2l-1)\big[\delta^{+}_{nm,l}(t) + \delta^{-}_{nm,l}(t)\big]\Big]; \forall n \in N; \forall t \in T \quad (4.24)$$

$$0 \le P^{\max}_{nm} - B_{nm}\sum_{l=1}^{L}\big[\delta^{+}_{nm,l}(t) - \delta^{-}_{nm,l}(t)\big]; \forall n \in N; \forall m \in \Omega_n; \forall t \in T \quad (4.25)$$

$$0 = P_{Gib}(t) - \widetilde{P}_{Gib}(t);\ \forall i \in G; b = 1, \dots, N_{Gi}; \forall t \in T \quad (4.26)$$

$$0 = P_{Djk}(t) - \widetilde{P}_{Djk}(t);\ \forall j \in D; k = 1, \dots, N_{Dj}; \forall t \in T \quad (4.27)$$

$$0 \le \Delta\delta - \delta^{+}_{nm,l}(t);\ \forall n \in N; \forall m \in \Omega_n; l = 1, \dots, L; \forall t \in T \quad (4.28)$$

$$0 \le \Delta\delta - \delta^{-}_{nm,l}(t);\ \forall n \in N; \forall m \in \Omega_n; l = 1, \dots, L; \forall t \in T \quad (4.29)$$

$$\delta^{+}_{nm,l}(t) \ge 0;\ \forall n \in N; \forall m \in \Omega_n; l = 1, \dots, L; \forall t \in T \quad (4.30)$$

$$\delta^{-}_{nm,l}(t) \ge 0;\ \forall n \in N; \forall m \in \Omega_n; l = 1, \dots, L; \forall t \in T \quad (4.31)$$

$$\gamma_{nm}(t) \ge 0;\ \forall n \in N; \forall m \in \Omega_n; \forall t \in T \quad (4.32)$$

$$\zeta^{+}_{nm,l}(t) \ge 0;\ \forall n \in N; \forall m \in \Omega_n; l = 1, \dots, L; \forall t \in T \quad (4.33)$$

$$\zeta^{-}_{nm,l}(t) \ge 0;\ \forall n \in N; \forall m \in \Omega_n; l = 1, \dots, L; \forall t \in T. \quad (4.34)$$

d) Fixed binary variables:

$$\overline{v}_i(t) = v_i^{(\nu)}(t)\ : \kappa_{vi}(t);\ \forall i \in G; \forall t \in T. \quad (4.35)$$

The first term of the objective function (4.1), Z_{QPP}, corresponds to the summation of all the inequality constraints of this problem multiplied by their respective dual variables. Note that Z_{QPP} should be zero at an optimal solution, corresponding to an actual MLCP solution.

The last equation, (4.35), forces the on / off status of the units to the values obtained through the master problem at the present iteration. Note that superscript ν is the iteration counter.

4.3.1.2 Master Problem

The master problem refines the on / off status for the generating units of the generating companies using information provided by the solutions to the subproblem, that is, the sensitivity of social welfare with respect to the on / off status of the units, $\kappa_{vi}(t)$. Additionally, fixed, start-up and shut-down costs are incorporated into the objective function of this problem because they only depend on binary variables. The master problem at iteration ν is defined by the following mixed-integer linear programming problem.

Minimize

$$\alpha + \sum_{t \in T} \sum_{i \in G} \left[C_{Gi}^{\text{fx}} v_i(t) + c_{Gi}^{\text{su}}(t) + c_{Gi}^{\text{sd}}(t) \right] \tag{4.36}$$

subject to

a) Benders cuts:

$$\alpha \geq Z_{\text{Sub}}^{(\ell)} + \sum_{t \in T} \sum_{i \in G} \left[\kappa_{vi}^{(\ell)}(t) \big(v_i(t) - v_i^{(\ell)}(t) \big) \right]; \; \ell = 1, \ldots, \nu - 1. \tag{4.37}$$

b) Start-up and shut-down cost constraints for the generating units:

$$c_{Gi}^{\text{su}}(t) \geq C_{Gi}^{\text{su}} \big[v_i(t) - v_i(t-1) \big]; \; \forall i \in G; \forall t \in T \tag{4.38}$$

$$c_{Gi}^{\text{su}}(t) \geq 0; \; \forall i \in G; \forall t \in T \tag{4.39}$$

$$c_{Gi}^{\text{sd}}(t) \geq C_{Gi}^{\text{sd}} \big[v_i(t-1) - v_i(t) \big]; \; \forall i \in G; \forall t \in T \tag{4.40}$$

$$c_{Gi}^{\text{sd}}(t) \geq 0; \; \forall i \in G; \forall t \in T. \tag{4.41}$$

c) Feasibility conditions:

$$\sum_{i \in G} v_i(t) P_{Gi}^{\max} \geq \sum_{j \in D} P_{Dj}^{\min}(t); \; \forall t \in T \tag{4.42}$$

$$\sum_{i \in G} v_i(t) P_{Gi}^{\min} \leq \sum_{j \in D} \sum_{k=1}^{N_{Dj}} P_{Djk}^{\max}(t); \; \forall t \in T \tag{4.43}$$

$$P_{Gi}^{\max} v_i(1) \geq \sum_{b=1}^{N_{Gi}} P_{Gib}(0) - R_i^{\text{dn}} v_i(1) - R_i^{\text{sd}} \big[1 - v_i(1) \big]; \; \forall i \in G. \tag{4.44}$$

d) Lower limit for α:

$$\alpha \geq \alpha^{\min}. \tag{4.45}$$

The objective function (4.36) includes α, which is a lower bound approximation of the objective function of the multi-period equilibrium problem, and fixed, start-up and shut-down costs. The set of constraints (4.37) are called the Benders cuts. These cuts provide information to the master problem to improve the on / off status decisions. Note that $Z_{\text{Sub}}^{(\ell)}$ represents the objective function value of the subproblem for each of the previous iterations. Constraints (4.38)-(4.41) state start-up and shut-down cost constraints of the generating units in each time period. Constraints (4.42), (4.43) and (4.44) force the master problem to generate solutions that satisfy the minimum demand requirements, the minimum power output of the units and the ramp rate limits at the first time period, respectively. These constraints ensure the feasibility of the subproblem. Finally, constraint (4.45) states a lower bound for α.

The solution of this problem defines the on / off status of each unit of each generating company in each time period, $v_i(t)$.

4.3.1.3 Bounds

The objective function value of the master problem, defined by equations (4.36)-(4.45), is a lower bound of the optimal objective function value of the multi-period equilibrium problem. This is so because problem (4.36)-(4.45) is a relaxation of the equilibrium problem. Therefore, for iteration ν, the optimal value of the objective function of the master problem is a lower bound of the optimal value of the objective function of the equilibrium problem.

$$Z_{\text{down}}^{(\nu)} = \alpha^{(\nu)} + \sum_{t\in T}\sum_{i\in G}\big[C_{Gi}^{\text{fx}} v_i^{(\nu)}(t) + c_{Gi}^{\text{su}(\nu)}(t) + c_{Gi}^{\text{sd}(\nu)}(t)\big]. \tag{4.46}$$

On the other hand, an upper bound of the optimal objective function value of the equilibrium problem is readily available because problem (4.1)-(4.35) is more constrained than the equilibrium problem. Therefore, an upper bound of the optimal value of the objective function of the equilibrium problem is

$$\begin{aligned} Z_{\text{up}}^{(\nu)} = Z_{\text{QPP}}^{(\nu)} &- \sum_{t\in T}\sum_{j\in D}\sum_{k=1}^{N_{Dj}} \lambda_{Djk}^{\text{B}}(t) P_{Djk}^{(\nu)}(t) + \sum_{t\in T}\sum_{i\in G}\sum_{b=1}^{N_{Gi}} \lambda_{Gib}^{\text{B}}(t) P_{Gib}^{(\nu)}(t) \\ &+ \sum_{t\in T}\sum_{i\in G}\big[C_{Gi}^{\text{fx}} v_i^{(\nu)}(t) + c_{Gi}^{\text{su}(\nu)}(t) + c_{Gi}^{\text{sd}(\nu)}(t)\big]. \end{aligned} \tag{4.47}$$

Note that the typical behavior of the lower bound of the optimal objective function value smoothly increases while the upper bound can either decrease or increase for each iteration. This behavior is checked in the numerical results of Chapter 5.

4.3.2 Solution Technique: The Benders Decomposition Algorithm

The multi-period equilibrium problem as stated in the previous subsection is a large-scale problem that includes continuous and binary variables and is defined and solved using the Benders decomposition method [7, 21, 43]. The master problem defines the on / off status for the generating units fixing the corresponding binary variables. The subproblem is a multi-period equilibrium problem with the binary variables fixed to given values by the master problem. In due order of succession, the master problem refines the on / off status for the generating units using the sensitivity of social welfare with respect to the value of the status variables defined in the master problem in the previous iteration. This iterative procedure continues until some cost tolerance is reached. That is:

a) Once binary variables are fixed to specified feasible values, the resulting continuous multi-period problem, the subproblem, is solved for its continuous variables.

b) Using marginal information obtained in a), Benders master problem allows improved values for the binary variables that were fixed in a) to be found.

c) The coordinated iteration of points a) and b) (Benders decomposition algorithm), allows a global optimum in both continuous and binary variables within the whole multi-period market horizon to be attained.

The algorithm to solve the multi-period equilibrium clearing without minimum profit conditions is formally described below.

Algorithm 4.1 (The Benders decomposition algorithm to solve the multi-period equilibrium without minimum profit conditions).

Step 0: Initialization. Initialize the iteration counter, $\nu = 1$.

Solve the initial mixed-integer linear programming master problem below that does not include Benders cuts.

Minimize

$$\alpha + \sum_{t \in T} \sum_{i \in G} \left[C^{\text{fx}}_{Gi} v_i(t) + c^{\text{su}}_{Gi}(t) + c^{\text{sd}}_{Gi}(t) \right] \tag{4.48}$$

subject to

$$c^{\mathrm{su}}_{Gi}(t) \geq C^{\mathrm{su}}_{Gi}\big[v_i(t) - v_i(t-1)\big];\ \forall i \in G; \forall t \in T \tag{4.49}$$

$$c^{\mathrm{su}}_{Gi}(t) \geq 0;\ \forall i \in G; \forall t \in T \tag{4.50}$$

$$c^{\mathrm{sd}}_{Gi}(t) \geq C^{\mathrm{sd}}_{Gi}\big[v_i(t-1) - v_i(t)\big];\ \forall i \in G; \forall t \in T \tag{4.51}$$

$$c^{\mathrm{sd}}_{Gi}(t) \geq 0;\ \forall i \in G; \forall t \in T \tag{4.52}$$

$$\sum_{i \in G} v_i(t) P^{\max}_{Gi} \geq \sum_{j \in D} P^{\min}_{Dj}(t);\ \forall t \in T \tag{4.53}$$

$$\sum_{i \in G} v_i(t) P^{\min}_{Gi} \leq \sum_{j \in D} \sum_{k=1}^{N_{Dj}} P^{\max}_{Djk}(t);\ \forall t \in T \tag{4.54}$$

$$P^{\max}_{Gi} v_i(1) \geq \sum_{b=1}^{N_{Gi}} P_{Gib}(0) - R^{\mathrm{dn}}_i v_i(1) - R^{\mathrm{sd}}_i\big[1 - v_i(1)\big];\ \forall i \in G \tag{4.55}$$

$$\alpha \geq \alpha^{\min}. \tag{4.56}$$

Its solution is $v_i^{(1)}(t)$ and $\alpha^{(1)} = \alpha^{\min}$.

Step 1: Subproblem solution. Solve the quadratic programming subproblem below.

Minimize

$$Z_{\mathrm{QPP}} - \sum_{t \in T} \sum_{j \in D} \sum_{k=1}^{N_{Dj}} \lambda^{\mathrm{B}}_{Djk}(t) P_{Djk}(t) + \sum_{t \in T} \sum_{i \in G} \sum_{b=1}^{N_{Gi}} \lambda^{\mathrm{B}}_{Gib}(t) P_{Gib}(t) \tag{4.57}$$

subject to

a) Optimality conditions of all the problems of the generating companies: constraints (4.2)-(4.13).

b) Optimality conditions of all the problems of the consumers: constraints (4.14)-(4.19).

c) Optimality conditions of the ISO problem: constraints (4.20)-(4.34).

d) Fixed binary variables:

$$\overline{v}_i(t) = v_i^{(\nu)}(t)\ \ : \kappa_{vi}(t);\ \forall i \in G; \forall t \in T. \tag{4.58}$$

Note that binary variables are fixed to the values obtained in the master problem.

Step 2: Convergence checking. Compute a lower bound of the optimal value of the objective function of the equilibrium problem,

$$Z^{(\nu)}_{\mathrm{down}} = \alpha^{(\nu)} + \sum_{t \in T} \sum_{i \in G} \big[C^{\mathrm{fx}}_{Gi} v_i^{(\nu)}(t) + c^{\mathrm{su}(\nu)}_{Gi}(t) + c^{\mathrm{sd}(\nu)}_{Gi}(t)\big], \tag{4.59}$$

and an upper bound of the optimal value of the objective function of the equilibrium problem,

$$Z_{\text{up}}^{(\nu)} = Z_{\text{QPP}}^{(\nu)} - \sum_{t\in T}\sum_{j\in D}\sum_{k=1}^{N_{Dj}} \lambda_{Djk}^{\text{B}}(t)P_{Djk}^{(\nu)}(t) + \sum_{t\in T}\sum_{i\in G}\sum_{b=1}^{N_{Gi}} \lambda_{Gib}^{\text{B}}(t)P_{Gib}^{(\nu)}(t) + \sum_{t\in T}\sum_{i\in G}\left[C_{Gi}^{\text{fx}}v_i^{(\nu)}(t) + c_{Gi}^{\text{su}(\nu)}(t) + c_{Gi}^{\text{sd}(\nu)}(t)\right]. \tag{4.60}$$

If $\left|Z_{\text{up}}^{(\nu)} - Z_{\text{down}}^{(\nu)}\right|$ is smaller than a pre-specified tolerance, that is,

$$\left|Z_{\text{QPP}}^{(\nu)} - \sum_{t\in T}\sum_{j\in D}\sum_{k=1}^{N_{Dj}} \lambda_{Djk}^{\text{B}}(t)P_{Djk}^{(\nu)}(t) + \sum_{t\in T}\sum_{i\in G}\sum_{b=1}^{N_{Gi}} \lambda_{Gib}^{\text{B}}(t)P_{Gib}^{(\nu)}(t) - \alpha^{(\nu)}\right| \leq \varepsilon, \tag{4.61}$$

stop, an optimal solution has been found. If this is not the case, the algorithm continues to the next step.

Step 3: Master problem solution. Update the iteration counter, $\nu \leftarrow \nu + 1$.

Solve the following mixed-integer linear programming master problem that includes Benders cuts.

Minimize

$$\alpha + \sum_{t\in T}\sum_{i\in G}\left[C_{Gi}^{\text{fx}}v_i(t) + c_{Gi}^{\text{su}}(t) + c_{Gi}^{\text{sd}}(t)\right] \tag{4.62}$$

subject to

$$\alpha \geq Z_{\text{sub}}^{(\ell)} + \sum_{t\in T}\sum_{i\in G}\left[\kappa_{vi}^{(\ell)}(t)\big(v_i(t) - v_i^{(\ell)}(t)\big)\right];\ \ell = 1,\ldots,\nu-1 \tag{4.63}$$

$$c_{Gi}^{\text{su}}(t) \geq C_{Gi}^{\text{su}}\big[v_i(t) - v_i(t-1)\big];\ \forall i \in G; \forall t \in T \tag{4.64}$$

$$c_{Gi}^{\text{su}}(t) \geq 0;\ \forall i \in G; \forall t \in T \tag{4.65}$$

$$c_{Gi}^{\text{sd}}(t) \geq C_{Gi}^{\text{sd}}\big[v_i(t-1) - v_i(t)\big];\ \forall i \in G; \forall t \in T \tag{4.66}$$

$$c_{Gi}^{\text{sd}}(t) \geq 0;\ \forall i \in G; \forall t \in T \tag{4.67}$$

$$\sum_{i\in G} v_i(t)P_{Gi}^{\max} \geq \sum_{j\in D} P_{Dj}^{\min}(t);\ \forall t \in T \tag{4.68}$$

$$\sum_{i\in G} v_i(t)P_{Gi}^{\min} \leq \sum_{j\in D}\sum_{k=1}^{N_{Dj}} P_{Djk}^{\max}(t);\ \forall t \in T \tag{4.69}$$

$$P_{Gi}^{\max}v_i(1) \geq \sum_{b=1}^{N_{Gi}} P_{Gib}(0) - R_i^{\text{dn}}v_i(1) - R_i^{\text{sd}}\big[1 - v_i(1)\big];\ \forall i \in G \tag{4.70}$$

$$\alpha \geq \alpha^{\min}. \tag{4.71}$$

The solution of this problem is $v_i^{(\nu)}(t)$ and $\alpha^{(\nu)}$. The algorithm continues in Step 1.

$\blacksquare$

Numerical simulations using different power systems show the appropriate convergence behavior of the solution; however, no formal proof of convergence has been developed.

4.3.3 Problem Size

The master problem is a mixed-integer linear programming problem whose numbers of variables and constraints are indicated in Table 4.1. The commercial solver CPLEX [40, 57] is an efficient tool to solve mixed-integer programming problems. This solver can be used under GAMS [14] or AMPL [36]. Analogously, the subproblem is a continuous quadratic programming problem whose size is shown in Table 4.1. This subproblem can be solved using the well-known commercial solvers MINOS [70] or CONOPT [31], which can work under GAMS [14, 39], AMPL [2, 36] and AIMMS [75].

Table 4.1: Size of problems

	Number of continuous variables	Number of binary variables	Number of constraints
Master problem	$1 + 2N_G N_T$	$N_G N_T$	$N_G(4N_T + 1)$ $+2N_T + \nu$
Subproblem	$4N_T(N_{GB} + N_{DB})$ $+N_T(4N_G + N_D)$ $+N_T(N_N + 2N_L)$ $+8N_T N_L L$	—	$6N_T(N_{GB} + N_{DB})$ $+N_T(8N_G + 2N_D)$ $+N_T(N_N + 4N_L)$ $+16N_T N_L L$

Note that N_T represents the number of time periods considered.

4.3.4 Example

The example in this section illustrates the multi-period equilibrium model for the 4-node network presented in Subsection 3.2.5, using the Benders decomposition algorithm to compute it.

4.3.4.1 Data

The time horizon considered consists of 3 time periods. The capacity limits of unit 1 and unit 2 are 200 MW and 400 MW, respectively; and the minimum power output is 20 MW and 50 MW for unit 1 and 2, respectively. The generating units bids at their marginal costs in each time period. Therefore, we suppose that the generating unit bids are the same for the three time periods and correspond to those shown in Table 3.2, Subsection 3.2.5.1. On the other hand, the minimum demand requirements considered for each node of the network in peak load condition during the market horizon corresponds to the values shown in Figure 3.2, Subsection 3.2.5.1, and demand bids at the time of system peak are shown in Table 3.3, Subsection 3.2.5.1. The hourly loads are obtained in percentages of the peak load. These percentages are provided in Table 4.2. The minimum demand requirements and the size of the demand bids for each time period are obtained modifying the peak load values according to these percentages. The price of the demand bids are considered to be the same in every time period and correspond to the bid prices shown in Table 3.3, Subsection 3.2.5.1.

Table 4.2: Hourly load in percentage of peak load. Example 4.3.4

Hour	Percentage of peak load [%]
1	67
2	100
3	83

The fixed, start-up and shut-down costs of each generating unit are provided in Table 4.3. This table also provides the ramp rate of each generating unit and the initial status of each unit. Note that we consider that ramping-up, ramping-down, start-up ramping and shut-down ramping limits coincide for the same unit.

Table 4.3: Generating unit data. Example 4.3.4

Unit	Fixed cost [$/h]	Start-up cost [$]	Shut-down cost [$]	Ramp rate limits [MW/h]	Initial power output [MW]
1	5	80	0	150	100
2	5	80	0	250	200

Topology and line data of the 4-node network can be found in Subsection 3.2.5.1.

4.3.4.2 Multi-Period Equilibrium

The multi-period equilibrium problem is solved using the Benders decomposition Algorithm 4.1 explained in Subsection 4.3.2. The solution is achieved in 6 iterations and 0.71 seconds of CPU within a relative tolerance lower than 0.001. The computer used is a Dell PowerEdge 6600 with 4 processors at 1.60 GHz and 2 GB of RAM memory.

Table 4.4 provides the power output, the revenue and the profit of each generating unit in each time period. The total profits of units 1 and 2 along the whole multi-period framework are $ 366 and $ 225, respectively. Note that profit for unit 2 in period 1 is negative, so this unit loses money in that period. Start-up costs along the market time horizon are zero for both generating units because no unit starts-up during the time horizon.

Table 4.4: Results for the generating units. Example 4.3.4

Period	Unit	Power output [MW]	Revenue [$/h]	Profit [$/h]
1	1	100.00	1920.00	55.00
	2	136.14	2613.87	-5.00
2	1	100.00	2023.34	158.34
	2	217.15	4343.06	115.00
3	1	100.00	2017.53	152.53
	2	193.14	3862.86	115.00

Table 4.5 presents results concerning power consumed and demand cost for each demand in each time period. Note that power consumed in period 2 is higher than in the other periods and, therefore, the demand costs are the highest.

Table 4.5: Results for the demands. Example 4.3.4

Period	Node	Power consumed [MW]	Demand cost [$/h]
1	3	167.50	3258.38
	4	67.00	1292.03
2	3	214.14	4389.86
	4	100.00	2032.19
3	3	207.50	4241.54
	4	83.00	1681.87

Locational marginal prices in each node and time period are different.

These prices are gathered together in Table 4.6. We can observe that the highest nodal price occurs at the period with the highest demand, period 2.

Table 4.6: Locational marginal prices. Example 4.3.4

Node	Period 1 [$/MWh]	Period 2 [$/MWh]	Period 3 [$/MWh]
1	19.20	20.23	20.18
2	19.20	20.00	20.00
3	19.45	20.50	20.44
4	19.28	20.32	20.26

4.4 Equilibrium Including Minimum Profit Constraints

This section analyzes the multi-period equilibrium of a pool-based electricity market that includes conditions for ensuring minimum profit levels for some of the generating units. These conditions are complex to model as they include both primal and dual variables. This problem is formulated using Benders decomposition [7, 21, 43]. In this case, the subproblem includes nonlinear constraints, the minimum profit conditions, which complicate its solution. A successive over-relaxation iterative method is applied to find the solution to this subproblem.

The multi-period equilibrium problem is analyzed below, and an adequate procedure to solve it is presented in this section.

4.4.1 Considerations on Minimum Profit Constraints

The reason for considering minimum profit conditions is justified in Chapter 3, subsection 3.3.1. For example, minimum profit conditions are used in some markets to promote generation capacity investments [73]. Therefore, there is a need to develop procedures that directly clear the market including such conditions. These procedures make it possible to avoid economical inefficiencies due to readjusting market results to meet the minimum profit conditions.

The minimum profit conditions declared by the generating units are more relevant in a multi-period framework. In this setting, the generating units alter their production schedules to meet the pre-specified minimum profit while the possible infeasibilities of the solution are minimized.

4.4.2 Formulation using Benders Decomposition

As stated in Subsection 4.3.1, the multi-period equilibrium can be obtained considering the set of the optimization problems corresponding to the generating companies, the consumers and the ISO. Some of these problems include continuous and binary variables, making it impossible to directly solve the equilibrium problem.

As in the case of no minimum profit constraints, this difficulty is overcame using the Benders decomposition technique [7, 21, 43] to formulate the market equilibrium problem. This technique decomposes the equilibrium problem into a subproblem and a master problem. The master problem is a mixed-integer linear programming problem whose target is to obtain the values of the binary variables. The binary variables are fixed to these values in the subproblem turning it into a large-scale nonlinear problem. The most important nonlinearity of the subproblem is due to the minimum profit conditions which are nonlinear inequalities. Therefore, the successive overrelaxation iterative method explained in Chapter 3, Subsection 3.3.4.2 is used to solve the subproblem.

Once the subproblem is solved, the master problem is solved again to refine the on / off status for the generating units using information provided by the subproblem.

The subproblem and the master problem are formulated in the following subsections.

4.4.2.1 Subproblem: Multi-Period Equilibrium for Fixed Status (Binary) Variables

The subproblem is a multi-period equilibrium problem with the binary variables fixed to given values. This problem is defined by the optimality conditions for the problems corresponding to the maximum profit / utility of the generating companies / consumers and the maximum social welfare of the ISO. We resort to complementarity theory [25] basically for two reasons. The first one is that it allows several conflicting problems to be solved at the same time. And the second one is that it allows constraints on dual variables to be included in the model, i.e., the minimum profit conditions which involve price variables.

The mixed linear complementarity problem determined by the optimality conditions for the problems of the market agents can be solved through an equivalent quadratic programming problem [25, 59]. This quadratic problem is extended to include the minimum profit constraints for the units that declare such a condition. As in the case that minimum profit conditions are not considered, Subsection 4.3.1.1, social welfare is subtracted from the objective function of the subproblem to improve computational behavior.

If minimum profit conditions are included as constraints to the subproblem, it turns into a nonlinearly constrained nonlinear programming problem with constraints involving both primal and dual variables. In Chapter 3, Subsection 3.3.4, we analyze three different solution techniques to solve these kinds of problems. The first method, proposes to directly solve the nonlinear problem. The second method, the linear approximation method, requires including binary variables and a considerable amount of constraints to the problem, thus avoiding nonlinearity but complicating the resulting quadratic programming problem. Finally, the third method, the successive over-relaxation iterative method, requires the iterative solution of a quadratic programming problem without including additional variables and constraints. Numerical simulations have shown that when the dimension of the subproblem is high, the convergence of the successive over-relaxation iterative method is better than the convergence of the linear approximation method, therefore we have focused on this iterative method to solve the subproblem.

The formulation of the subproblem is stated below.

Minimize

$$Z_{\text{QPP}} - \sum_{t\in T}\sum_{j\in D}\sum_{k=1}^{N_{Dj}} \lambda^{\text{B}}_{Djk}(t) P_{Djk}(t) + \sum_{t\in T}\sum_{i\in G}\sum_{b=1}^{N_{Gi}} \lambda^{\text{B}}_{Gib}(t) P_{Gib}(t) \tag{4.72}$$

subject to

a) Optimality conditions of all problems of the generating companies:

$$0 \le \lambda^{\text{C}}_{Gib}(t) - \rho_{n(i)}(t) + \alpha_i(t) - \beta_i(t) + \phi_{ib}(t) + \tau_i(t) - \tau_i(t-1) \tag{4.73}$$

$$+ \psi_i(t-1) - \psi_i(t);\ \forall i \in G; b = 1,\ldots,N_{Gi}; \forall t \in T \tag{4.74}$$

$$0 \le P^{\max}_{Gi}\overline{v}_i(t) - \sum_{b=1}^{N_{Gi}} P_{Gib}(t);\ \forall i \in G; \forall t \in T \tag{4.75}$$

$$0 \le \sum_{b=1}^{N_{Gi}} P_{Gib}(t) - P^{\min}_{Gi}\overline{v}_i(t);\ \forall i \in G; \forall t \in T \tag{4.76}$$

$$0 \le P^{\max}_{Gib}(t) - P_{Gib}(t);\ \forall i \in G; b = 1,\ldots,N_{Gi}-1; \forall t \in T \tag{4.77}$$

$$\begin{aligned} 0 \le{}& R^{\text{up}}_i\overline{v}_i(t-1) + R^{\text{su}}_i\big[\overline{v}_i(t) - \overline{v}_i(t-1)\big] + P^{\max}_{Gi}\big[1 - \overline{v}_i(t)\big] \\ &- \sum_{b=1}^{N_{Gi}} P_{Gib}(t) + \sum_{b=1}^{N_{Gi}} P_{Gib}(t-1);\ \forall i \in G; \forall t \in T \end{aligned} \tag{4.78}$$

$$\begin{aligned} 0 \le{}& R^{\text{dn}}_i\overline{v}_i(t) + R^{\text{sd}}_i\big[\overline{v}_i(t-1) - \overline{v}_i(t)\big] + P^{\max}_{Gi}\big[1 - \overline{v}_i(t-1)\big] \\ &- \sum_{b=1}^{N_{Gi}} P_{Gib}(t-1) + \sum_{b=1}^{N_{Gi}} P_{Gib}(t);\ \forall i \in G; \forall t \in T \end{aligned} \tag{4.79}$$

$$P_{Gib}(t) \ge 0;\ \forall i \in G; b = 1,\ldots,N_{Gi}; \forall t \in T \tag{4.80}$$

$$\alpha_i(t) \ge 0;\ \forall i \in G; \forall t \in T \tag{4.81}$$

$$\beta_i(t) \ge 0;\ \forall i \in G; \forall t \in T \tag{4.82}$$

$$\phi_{ib}(t) \ge 0;\ \forall i \in G; b = 1,\ldots,N_{Gi}; \forall t \in T \tag{4.83}$$

$$\tau_i(t) \geq 0;\ \forall i \in G; \forall t \in T \tag{4.84}$$

$$\psi_i(t) \geq 0;\ \forall i \in G; \forall t \in T. \tag{4.85}$$

b) Optimality conditions of all problems of the consumers:

$$0 \leq \rho_{n(j)}(t) - \lambda^{\mathrm{U}}_{Djk}(t) - \sigma_j(t) + \varphi_{jk}(t); \forall j \in D; k = 1, \ldots, N_{Dj}; \forall t \in T \tag{4.86}$$

$$0 \leq \sum_{k=1}^{N_{Dj}} P_{Djk}(t) - P^{\min}_{Dj}(t);\ \forall j \in D; \forall t \in T \tag{4.87}$$

$$0 \leq P^{\max}_{Djk}(t) - P_{Djk}(t);\ \forall j \in D; k = 1, \ldots, N_{Dj}; \forall t \in T \tag{4.88}$$

$$P_{Djk}(t) \geq 0;\ \forall j \in D; k = 1, \ldots, N_{Dj}; \forall t \in T \tag{4.89}$$

$$\sigma_j(t) \geq 0;\ \forall j \in D; \forall t \in T \tag{4.90}$$

$$\varphi_{jk}(t) \geq 0;\ \forall j \in D; k = 1, \ldots, N_{Dj}; \forall t \in T. \tag{4.91}$$

c) Optimality conditions of the ISO problem:

$$0 = \lambda^{\mathrm{B}}_{Gib}(t) - \rho_{n(i)}(t) + \mu_{Gib}(t);\ \forall i \in G; b = 1, \ldots, N_{Gi}; \forall t \in T \tag{4.92}$$

$$0 = \rho_{n(j)}(t) - \lambda^{\mathrm{B}}_{Djk}(t) + \nu_{Djk}(t);\ \forall j \in D; k = 1, \ldots, N_{Dj}; \forall t \in T \tag{4.93}$$

$$0 \leq \rho_n(t)\big[B_{nm} - \frac{1}{2}G_{nm}\Delta\delta(2l-1)\big] + B_{nm}\ \gamma_{nm}(t) + \zeta^{+}_{nm,l}(t); \forall n \in N; \forall m \in \Omega_n; l = 1, \ldots, L; \forall t \in T \tag{4.94}$$

$$0 \leq \rho_n(t)\big[-B_{nm} - \frac{1}{2}G_{nm}\Delta\delta(2l-1)\big] - B_{nm}\ \gamma_{nm}(t) + \zeta^{-}_{nm,l}(t); \forall n \in N; \forall m \in \Omega_n; l = 1, \ldots, L; \forall t \in T \tag{4.95}$$

$$\begin{aligned} 0 = &- \sum_{i \in \theta_n} \sum_{b=1}^{N_{Gi}} \widetilde{P}_{Gib}(t) + \sum_{j \in \vartheta_n} \sum_{k=1}^{N_{Dj}} \widetilde{P}_{Djk}(t) \\ &+ \sum_{m \in \Omega_n} B_{nm} \sum_{l=1}^{L} \big[\delta^{+}_{nm,l}(t) - \delta^{-}_{nm,l}(t)\big] \\ &- \frac{1}{2} \sum_{m \in \Omega_n} \Big[G_{nm}\Delta\delta \sum_{l=1}^{L}(2l-1)\big[\delta^{+}_{nm,l}(t) + \delta^{-}_{nm,l}(t)\big]\Big]; \\ &\forall n \in N; \forall t \in T \end{aligned} \tag{4.96}$$

$$0 \leq P^{\max}_{nm} - B_{nm} \sum_{l=1}^{L} \big[\delta^{+}_{nm,l}(t) - \delta^{-}_{nm,l}(t)\big]; \forall n \in N; \forall m \in \Omega_n; \forall t \in T \tag{4.97}$$

$$0 = P_{Gib}(t) - \widetilde{P}_{Gib}(t);\ \forall i \in G; b = 1, \ldots, N_{Gi}; \forall t \in T \tag{4.98}$$

$$0 = P_{Djk}(t) - \widetilde{P}_{Djk}(t);\ \forall j \in D; k = 1, \ldots, N_{Dj}; \forall t \in T \tag{4.99}$$

$$0 \leq \Delta\delta - \delta^{+}_{nm,l}(t);\ \forall n \in N; \forall m \in \Omega_n; l = 1, \ldots, L; \forall t \in T \tag{4.100}$$

$$0 \leq \Delta\delta - \delta^{-}_{nm,l}(t);\ \forall n \in N; \forall m \in \Omega_n; l = 1, \ldots, L; \forall t \in T \tag{4.101}$$

$$\delta^{+}_{nm,l}(t) \geq 0;\ \forall n \in N; \forall m \in \Omega_n; l = 1, \ldots, L; \forall t \in T \tag{4.102}$$

$$\delta^{-}_{nm,l}(t) \geq 0;\ \forall n \in N; \forall m \in \Omega_n; l = 1, \ldots, L; \forall t \in T \tag{4.103}$$

$$\gamma_{nm}(t) \geq 0;\ \forall n \in N; \forall m \in \Omega_n; \forall t \in T \tag{4.104}$$

$$\zeta^{+}_{nm,l}(t) \geq 0;\ \forall n \in N; \forall m \in \Omega_n; l = 1, \ldots, L; \forall t \in T \tag{4.105}$$

$$\zeta^{-}_{nm,l}(t) \geq 0;\ \forall n \in N; \forall m \in \Omega_n; l = 1, \ldots, L; \forall t \in T. \tag{4.106}$$

d) Minimum profit conditions:

$$\sum_{t \in T} \sum_{b=1}^{N_{Gi}} \big(\rho_{n(i)}(t) - \lambda^{\mathrm{C}}_{Gib}(t)\big) P_{Gib}(t) - \sum_{t \in T} \big[C^{\mathrm{fx}}_{Gi} \overline{v}_i(t) + c^{\mathrm{su}}_{Gi}(t) + c^{\mathrm{sd}}_{Gi}(t)\big] \geq K_i; \forall i \in G^{\mathrm{Mon}}. \tag{4.107}$$

e) Fixed binary variables:

$$\overline{v}_i(t) = v_i^{(\nu)}(t) \ : \kappa_{vi}(t);\ \forall i \in G; \forall t \in T. \tag{4.108}$$

The last equation, (4.108), forces the on / off status of the units to the values obtained from the master problem in the present iteration.

4.4.2.2 Master Problem

The master problem provides the values of the binary variables corresponding to the on / off status for the generating units. Information obtained solving the subproblem allows formulating a more accurate master problem that refines the on / off status values for the units. This information is considered in the master problem through the Benders cuts. The formulation of the master problem corresponds to the problem (4.36)-(4.45) whose constraints include: Benders cuts, start-up and shut-down cost constraints for the generating units, feasibility conditions and the lower bound for α.

The solution to this problem defines the on / off status of each unit of each generating company in each time period, $v_i(t)$.

4.4.2.3 Bounds

An upper bound of the optimal objective function value is readily available because the subproblem, (4.72)-(4.108), is more constrained than the multi-period equilibrium problem including minimum profit conditions. And a lower bound of the optimal objective function value of the equilibrium problem corresponds to the optimal objective function value of the master problem, (4.36)-(4.45), because this is a relaxation of the market equilibrium problem. These bounds are represented by the following equations:

$$Z_{\text{down}}^{(\nu)} = \alpha^{(\nu)} + \sum_{t\in T}\sum_{i\in G}\left[C_{Gi}^{\text{fx}} v_i^{(\nu)}(t) + c_{Gi}^{\text{su}(\nu)}(t) + c_{Gi}^{\text{sd}(\nu)}(t)\right] \tag{4.109}$$

$$Z_{\text{up}}^{(\nu)} = Z_{\text{QPP}}^{(\nu)} - \sum_{t\in T}\sum_{j\in D}\sum_{k=1}^{N_{Dj}} \lambda_{Djk}^{\text{B}}(t) P_{Djk}^{(\nu)}(t) + \sum_{t\in T}\sum_{i\in G}\sum_{b=1}^{N_{Gi}} \lambda_{Gib}^{\text{B}}(t) P_{Gib}^{(\nu)}(t)$$
$$+ \sum_{t\in T}\sum_{i\in G}\left[C_{Gi}^{\text{fx}} v_i^{(\nu)}(t) + c_{Gi}^{\text{su}(\nu)}(t) + c_{Gi}^{\text{sd}(\nu)}(t)\right]. \tag{4.110}$$

Note that the upper and lower bounds of the optimal objective function value are the same as those presented in the case in which minimum profit conditions are not considered, equations (4.46) and (4.47).

4.4.3 Uniqueness, Multiple Dual Solutions and Infeasibility

If the market is balanced using a single-period equilibrium, the only possibilities to attain a solution to meet minimum profit requirements involve altering locational marginal prices or the productions of the units. In some cases, this modification results in slight infeasibilities, as was explained in Chapter 3, Subsection 3.3.3. In such situation, the equilibrium becomes a near-equilibrium [65].

If we balance the market using a multi-period equilibrium as explained in the current chapter, there is an additional possibility to meet the minimum profit conditions imposed by the units. This consists in reorganizing the unit schedules, in such a way as to minimize or to eliminate the infeasibility.

In what follows, we discuss the effect of imposing minimum profit conditions on the multi-period equilibrium problem. There are two cases that are dealt with below.

4.4.3.1 Multiple Dual Solutions Case

The multi-period equilibrium problem might have multiple dual solutions and therefore have multiple prices for any given time period. This case is illustrated in Figure 4.3(a) for a given time period. Note that there is a range of prices at which the power supplied is equal to the power demanded. In this situation, a minimum profit-constrained, multi-period equilibrium problem generally results in a feasible problem whose optimal solution meets minimum profit conditions.

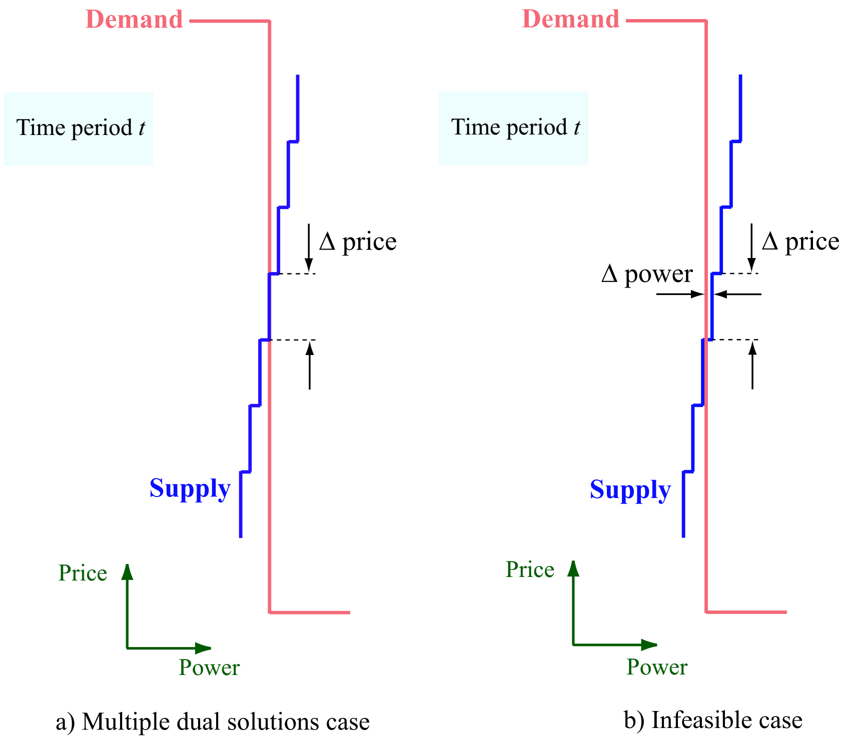

Figure 4.3: Multiple dual solutions and infeasible cases

4.4.3.2 Infeasible Case

The multi-period equilibrium problem has a unique solution in prices, generations, demands and flows. By adding minimum profit constraints to this problem, we simply create infeasibilities, assuming that the minimum profit condition is not attained for this unique solution beforehand. However, it should be noted that for practical applications, these infeasibilities are generally negligible as we reason in Chapter 3, Subsection 3.3.3.2. The conclusion is that the multi-period equilibrium satisfying minimum profit conditions is often in the vicinity of these "near-degeneracy" regions, in which there is a unique multi-period equilibrium albeit with step supply and demand curves as illustrated in Figure 4.3(b). In this figure, it can be observed that small increments in power (which create slight infeasibilities) result in significant price differences (which allow minimum profit conditions to be easily met).

Slight infeasibilities cause an optimal objective function value, Z_{QPP}, to be slightly different from zero in the subproblem. These slight infeasibilities are related to prices because power balance is enforced at every node and in every time period. The cost incurred due to price infeasibilities can be allocated pro-rata among market participants.

Considering the discussion above, we define a near-equilibrium as an optimal solution of a minimum profit-constrained, multi-period equilibrium problem that results in an optimal objective function value, Z_{QPP}, slightly different from zero. This implies that one or more of the complementarity conditions of the equilibrium problem are not fully satisfied.

Thus, in general, the optimal solution of the multi-period equilibrium problem represents a near-equilibrium that may include small complementary infeasibilities of negligible practical significance. An appropriate metric for the importance of such infeasibilities is the optimal value of the objective function, Z_{QPP}. The closer to zero this value is, the closer to feasibility the minimum profit-constrained, multi-period equilibrium problem is.

4.4.3.3 Infeasibility Cost

As we describe in Chapter 3, Subsection 3.3.3.3, the infeasibility of the multi-period equilibrium can be partly originated by the fact that a generating unit is paid at a price lower than its cost, or that a demand pay a price higher than its utility. The sum of these economic losses is called infeasibility cost.

We compensate each generating unit and demand with those economic losses through an uplift, and the total costs of these uplifts, that corresponds to the infeasibility cost, are paid by all market participants. Half of the infeasibility cost is allocated pro-rata among generating units, and the other half is allocated pro-rata among demands, as stated by the following equations,

$$C_i = \frac{C}{2} \frac{\displaystyle\sum_{t\in T}\sum_{b=1}^{N_{Gi}} P_{Gib}(t)}{\displaystyle\sum_{t\in T}\sum_{i=1}^{N_G}\sum_{b=1}^{N_{Gi}} P_{Gib}(t)}; \ \forall i \in G; \forall i \notin G^{\text{uplift}} \tag{4.111}$$

$$C_i = \frac{C}{2} \frac{\displaystyle\sum_{t\in T}\sum_{b=1}^{N_{Gi}} P_{Gib}(t)}{\displaystyle\sum_{t\in T}\sum_{i=1}^{N_G}\sum_{b=1}^{N_{Gi}} P_{Gib}(t)} - \text{Uplift}_i; \ \forall i \in G^{\text{uplift}} \tag{4.112}$$

$$C_j = \frac{C}{2} \frac{\displaystyle\sum_{t\in T}\sum_{k=1}^{N_{Dj}} P_{Djk}(t)}{\displaystyle\sum_{t\in T}\sum_{j=1}^{N_D}\sum_{k=1}^{N_{Dj}} P_{Djk}(t)}; \ \forall j \in D; \forall i \notin D^{\text{uplift}} \tag{4.113}$$

$$C_j = \frac{C}{2} \frac{\displaystyle\sum_{t\in T}\sum_{k=1}^{N_{Dj}} P_{Djk}(t)}{\displaystyle\sum_{t\in T}\sum_{j=1}^{N_D}\sum_{k=1}^{N_{Dj}} P_{Djk}(t)} - \text{Uplift}_j; \ \forall i \in D^{\text{uplift}}. \tag{4.114}$$

Note that the additional cost, C_i, is subtracted from the corresponding profit of the generating unit i, and the cost, C_j, is added to the corresponding demand cost of demand j.

4.4.4 Solution Technique: The Benders Decomposition Algorithm

The proposed technique to solve the multi-period equilibrium problem with minimum profit conditions is described below.

4.4.4.1 Benders Algorithm

The multi-period equilibrium problem as stated in Subsection 4.4.2 is a large-scale problem that includes continuous and binary variables and is defined and solved using the Benders decomposition method [7, 21, 43]. The master problem defines the on / off status for the generating units by solving for the corresponding binary variables. The subproblem is a multi-period equilibrium problem with the binary variables fixed to given values by the master problem. In turn, the master problem refines the on / off status for the generating units using the sensitivity of social welfare with respect to the value of the status variables defined in the master problem in the previous iteration. This iterative procedure continues until some cost tolerance is reached. The formal steps of the algorithm are described below.

Algorithm 4.2 (The Benders decomposition algorithm to solve the multi-period market equilibrium problem with minimum profit conditions).
Step 0: Initialization. Initialize the iteration counter, $\nu = 1$.

Solve the initial mixed-integer linear programming master problem below which does not include Benders cuts.

Minimize

$$\alpha + \sum_{t\in T}\sum_{i\in G}\left[C_{Gi}^{\text{fx}} v_i(t) + c_{Gi}^{\text{su}}(t) + c_{Gi}^{\text{sd}}(t)\right] \tag{4.115}$$

subject to

$$c_{Gi}^{\text{su}}(t) \geq C_{Gi}^{\text{su}}\left[v_i(t) - v_i(t-1)\right];\ \forall i \in G; \forall t \in T \tag{4.116}$$

$$c_{Gi}^{\text{su}}(t) \geq 0;\ \forall i \in G; \forall t \in T \tag{4.117}$$

$$c_{Gi}^{\text{sd}}(t) \geq C_{Gi}^{\text{sd}}\left[v_i(t-1) - v_i(t)\right];\ \forall i \in G; \forall t \in T \tag{4.118}$$

$$c_{Gi}^{\text{sd}}(t) \geq 0;\ \forall i \in G; \forall t \in T \tag{4.119}$$

$$\sum_{i\in G} v_i(t) P_{Gi}^{\max} \geq \sum_{j\in D} P_{Dj}^{\min}(t);\ \forall t \in T \tag{4.120}$$

$$\sum_{i\in G} v_i(t) P_{Gi}^{\min} \leq \sum_{j\in D}\sum_{k=1}^{N_{Dj}} P_{Djk}^{\max}(t);\ \forall t \in T \tag{4.121}$$

$$P_{Gi}^{\max} v_i(1) \geq \sum_{b=1}^{N_{Gi}} P_{Gib}(0) - R_i^{\mathrm{dn}} v_i(1) - R_i^{\mathrm{sd}} \big[1 - v_i(1)\big];\ \forall i \in G \tag{4.122}$$

$$\alpha \geq \alpha^{\min}. \tag{4.123}$$

Its solution is $v_i^{(1)}(t)$ and $\alpha^{(1)} = \alpha^{\min}$.

Step 1: Subproblem solution. Once binary variables are fixed to specified feasible values, the resulting continuous multi-period equilibrium problem, that is, the subproblem, is solved for its continuous variables. This subproblem is a nonlinear program and is solved using one of the three procedures indicated in Subsection 4.4.4.2.

Step 2: Convergence checking. Compute a lower bound of the optimal value of the objective function of the equilibrium problem,

$$Z_{\mathrm{down}}^{(\nu)} = \alpha^{(\nu)} + \sum_{t \in T} \sum_{i \in G} \big[C_{Gi}^{\mathrm{fx}} v_i^{(\nu)}(t) + c_{Gi}^{\mathrm{su}(\nu)}(t) + c_{Gi}^{\mathrm{sd}(\nu)}(t)\big], \tag{4.124}$$

and an upper bound of the optimal value of the objective function of the equilibrium problem,

$$\begin{aligned} Z_{\mathrm{up}}^{(\nu)} = Z_{\mathrm{QPP}}^{(\nu)} &- \sum_{t \in T} \sum_{j \in D} \sum_{k=1}^{N_{Dj}} \lambda_{Djk}^{\mathrm{B}}(t) P_{Djk}^{(\nu)}(t) + \sum_{t \in T} \sum_{i \in G} \sum_{b=1}^{N_{Gi}} \lambda_{Gib}^{\mathrm{B}}(t) P_{Gib}^{(\nu)}(t) \\ &+ \sum_{t \in T} \sum_{i \in G} \big[C_{Gi}^{\mathrm{fx}} v_i^{(\nu)}(t) + c_{Gi}^{\mathrm{su}(\nu)}(t) + c_{Gi}^{\mathrm{sd}(\nu)}(t)\big]. \end{aligned} \tag{4.125}$$

If $\big|Z_{\mathrm{up}}^{(\nu)} - Z_{\mathrm{down}}^{(\nu)}\big|$ is smaller than a pre-specified tolerance, that is,

$$\begin{aligned} \Bigg| Z_{\mathrm{QPP}}^{(\nu)} &- \sum_{t \in T} \sum_{j \in D} \sum_{k=1}^{N_{Dj}} \lambda_{Djk}^{\mathrm{B}}(t) P_{Djk}^{(\nu)}(t) \\ &+ \sum_{t \in T} \sum_{i \in G} \sum_{b=1}^{N_{Gi}} \lambda_{Gib}^{\mathrm{B}}(t) P_{Gib}^{(\nu)}(t) - \alpha^{(\nu)} \Bigg| \leq \varepsilon, \end{aligned} \tag{4.126}$$

stop, an optimal solution has been found. If this is not the case, the algorithm continues to the next step.

Step 3: Master problem solution. Update the iteration counter, $\nu \leftarrow \nu + 1$.

Using marginal information obtained in Step 1, the master problem finds improved values for the binary variables fixed in Step 1. Therefore, we must solve the following mixed-integer linear programming master problem.

Minimize

$$\alpha + \sum_{t \in T} \sum_{i \in G} \big[C_{Gi}^{\mathrm{fx}} v_i(t) + c_{Gi}^{\mathrm{su}}(t) + c_{Gi}^{\mathrm{sd}}(t)\big] \tag{4.127}$$

subject to

$$\alpha \geq Z_{\text{sub}}^{(\ell)} + \sum_{t \in T} \sum_{i \in G} \Big[\kappa_{vi}^{(\ell)}(t) \big(v_i(t) - v_i^{(\ell)}(t) \big) \Big]; \; \ell = 1, \ldots, \nu - 1 \tag{4.128}$$

$$c_{Gi}^{\text{su}}(t) \geq C_{Gi}^{\text{su}} \big[v_i(t) - v_i(t-1) \big]; \; \forall i \in G; \forall t \in T \tag{4.129}$$

$$c_{Gi}^{\text{su}}(t) \geq 0; \; \forall i \in G; \forall t \in T \tag{4.130}$$

$$c_{Gi}^{\text{sd}}(t) \geq C_{Gi}^{\text{sd}} \big[v_i(t-1) - v_i(t) \big]; \; \forall i \in G; \forall t \in T \tag{4.131}$$

$$c_{Gi}^{\text{sd}}(t) \geq 0; \; \forall i \in G; \forall t \in T \tag{4.132}$$

$$\sum_{i \in G} v_i(t) P_{Gi}^{\max} \geq \sum_{j \in D} P_{Dj}^{\min}(t); \; \forall t \in T \tag{4.133}$$

$$\sum_{i \in G} v_i(t) P_{Gi}^{\min} \leq \sum_{j \in D} \sum_{k=1}^{N_{Dj}} P_{Djk}^{\max}(t); \; \forall t \in T \tag{4.134}$$

$$P_{Gi}^{\max} v_i(1) \geq \sum_{b=1}^{N_{Gi}} P_{Gib}(0) - R_i^{\text{dn}} v_i(1) - R_i^{\text{sd}} \big[1 - v_i(1) \big]; \; \forall i \in G \tag{4.135}$$

$$\alpha \geq \alpha^{\min}. \tag{4.136}$$

The solution of this problem is $v_i^{(\nu)}(t)$ and $\alpha^{(\nu)}$. The algorithm continues in Step 1.

■

This algorithm allows an optimum in both continuous and binary variables to be attained within the whole multi-period market horizon. Numerical simulations using different power systems show the appropriate convergence behavior of the solution; however, no formal proof of convergence has been developed.

4.4.4.2 Solution of the subproblem

The subproblem is a nonlinearly constrained nonlinear problem defined by equations (4.72)-(4.108) and difficult to solve. The main difficulty lies in the nonlinearity of the minimum profit conditions. Three alternative procedures can be used to solve this problem.

a) To directly solve the subproblem using an appropriate nonlinear solver.

b) To linearize the nonlinear minimum profit constraints using Schur's decomposition and binary variables as stated in Subsection 3.3.4.1, [38, 41], and to solve the resulting mixed-integer quadratic problem.

c) To use the inner algorithm stated below, which is based on a successive over-relaxation method [42]. The description of this method is as follows.

Step 1.0: Initialization. Initialize the iteration counter of the inner iteration, $\eta = 1$. Without considering the minimum profit conditions, the subproblem is solved. This problem is formulated below.

Minimize

$$Z_{\text{QPP}} - \sum_{t\in T}\sum_{j\in D}\sum_{k=1}^{N_{Dj}} \lambda^{\text{B}}_{Djk}(t)P_{Djk}(t) + \sum_{t\in T}\sum_{i\in G}\sum_{b=1}^{N_{Gi}} \lambda^{\text{B}}_{Gib}(t)P_{Gib}(t) \quad (4.137)$$

subject to

(a) Optimality conditions of all problems of the generating companies: constraints (4.73)-(4.85).

(b) Optimality conditions of all problems of the consumers: constraints (4.86)-(4.91).

(c) Optimality conditions of the ISO problem: constraints (4.92)-(4.106).

(d) Fixed binary variables:

$$\overline{v}_i(t) = v_i^{(\nu)}(t) \ : \kappa_{vi}(t);\ \forall i \in G; \forall t \in T. \quad (4.138)$$

If the solution to this problem satisfies all the minimum profit requirements imposed by all generating units, the solution to the subproblem has been attained and the successive over-relaxation method concludes. Otherwise (any minimum profit requirement is violated), the method continues.

The solution to this problem is used to compute an initial estimate of the generating powers. That is,

$$\hat{P}^{(1)}_{Gib}(t) = a\bar{P}^{(1)}_{Gib}(t);\ \forall i \in G^{\text{M}^{\text{on}}}; \forall t \in T \quad (4.139)$$

where $\hat{P}^{(1)}_{Gib}(t)$ represents the initial estimate of the generating power of block b of unit i in hour t; $\bar{P}^{(1)}_{Gib}(t)$ is the optimal generating power value of block b of the generating unit i in hour t for the subproblem (4.137)-(4.138); and $a \geq 1$ is a constant.

Step 1.1: Subproblem including minimum profit conditions. The generating power values appearing in the minimum profit conditions are fixed to the corresponding estimated values. Therefore, the minimum profit conditions turn into linear expressions and the subproblem becomes a quadratic program that is solved. The minimum profit conditions considered in the subproblem have the form

$$\sum_{t\in T}\sum_{b=1}^{N_{Gi}}\big(\rho_{n(i)}(t)-\lambda^{\mathrm{C}}_{Gib}(t)\big)\hat{P}^{(\eta)}_{Gib}(t) - \sum_{t\in T}\big[C^{\mathrm{fx}}_{Gi}\bar{v}_i(t)+c^{\mathrm{su}}_{Gi}(t)+c^{\mathrm{sd}}_{Gi}(t)\big] \geq K_i;\ \forall i\in G^{\mathrm{M^{on}}}. \tag{4.140}$$

These constraints are only imposed for the generating units that declare such a condition and that remain on-line during at least a time period on the market horizon, $G^{\mathrm{M^{on}}}$. Minimum profit conditions are included as constraints to the quadratic problem presented in Step 1.0. The optimal generating power values for this problem are $\bar{P}^{(\eta+1)}_{Gib}(t)$.

Step 1.2: Generating power estimate updating. Update the estimates of the generating powers through the equation

$$\hat{P}^{(\eta+1)}_{Gib}(t) = d\bar{P}^{(\eta+1)}_{Gib}(t) + (1-d)\hat{P}^{(\eta)}_{Gib}(t);\ \forall i\in G^{\mathrm{M^{on}}};\forall t\in T \tag{4.141}$$

where the constant $d \in (0,1)$. Note that constant d does not change with each iteration.

If for all $i \in G^{\mathrm{M^{on}}}$, $\sum_{t\in T}\left|\dfrac{\hat{P}^{(\eta+1)}_{Gib}(t)-\hat{P}^{(\eta)}_{Gib}(t)}{\hat{P}^{(\eta)}_{Gib}(t)}\right| \leq \epsilon$, stop, the solution has been found and corresponds to the solution of Step 1.1; the inner algorithm concludes and the procedure continues in Step 2 of the Benders algorithm. If this is not the case, the iteration counter is updated, $\eta \leftarrow \eta + 1$ and the algorithm continues in Step 1.1.

Note that ϵ is an appropriate convergence tolerance.

■

From an experimental point of view, this successive over-relaxation algorithm presents good convergence behavior. A characterization of its convergence characteristic can be constructed based on results reported in [23, 45, 72, 79].

When the dimensions of the equilibrium problem are large, the successive over-relaxation method converges faster than the other two algorithms reported in a) and b). Therefore, for the multi-period case, it is recommended to use the successive over-relaxation algorithm.

4.4.5 Problem size

The master problem is a mixed-integer linear programming problem whose numbers of variables and constraints are indicated in Table 4.7. Analogously, the subproblem is a nonlinear programming problem, which is solved through a successive over-relaxation method that solves a quadratic programming problem in each iteration. The size of this quadratic programming problem is shown in Table 4.7. The master problem and the subproblem can be solved using the commercial solvers mentioned in Subsection 4.3.3.

Table 4.7: Size of problems

	Number of continuous variables	Number of binary variables	Number of constraints
Master problem	$1 + 2N_G N_T$	$N_G N_T$	$N_G(4N_T + 1)$ $+2N_T + \nu$
Subproblem	$4N_T(N_{GB} + N_{DB})$ $+N_T(4N_G + N_D)$ $+N_T(N_N + 2N_L)$ $+8N_T N_L L$	—	$6N_T(N_{GB} + N_{DB})$ $+N_T(8N_G + 2N_D)$ $+N_T(N_N + 4N_L)$ $+16N_T N_L L + N_{G^{\mathrm{Mon}}}$

Note that $N_{G^{\mathrm{Mon}}}$ represents the number of generating units that impose minimum profit conditions and remain on-line during at least one time period on the market horizon.

4.5 Economic Efficiency Metrics

The producer surplus, the consumer surplus, the social welfare and the merchandising surplus for a multi-period equilibrium are defined below.

The producer surplus is defined as the difference between what generating companies actually receive for selling the power and the minimum amount that they would have to receive in order to supply the given level of power output for the whole multi-period framework. Producer surplus can be computed using the following equation:

$$\mathrm{PS} = \sum_{t \in T} \sum_{i \in G} \sum_{b=1}^{N_{Gi}} \Big[\big(\rho_{n(i)}(t) - \lambda^{\mathrm{C}}_{Gib}(t)\big) P_{Gib}(t) - C^{\mathrm{fx}}_{Gi} v_i(t) - c^{\mathrm{su}}_{Gi}(t) - c^{\mathrm{sd}}_{Gi}(t) \Big]. \tag{4.142}$$

The consumer surplus is defined as the difference between what consumers are willing to pay for the power they buy and the amount that consumers actually pay for the whole multi-period framework. The following equation represents the consumer surplus:

$$\mathrm{CS} = \sum_{t \in T} \sum_{j \in D} \sum_{k=1}^{N_{Dj}} \big(\lambda^{\mathrm{U}}_{Djk}(t) - \rho_{n(j)}(t)\big) P_{Djk}(t). \tag{4.143}$$

The merchandising surplus is defined as the difference between the total demand costs and the revenues of all the generating units for the whole multi-period framework. The merchandising surplus is expressed by the following equation:

$$\mathrm{MS} = \sum_{t \in T} \sum_{j \in D} \sum_{k=1}^{N_{Dj}} \rho_{n(j)}(t) P_{Djk}(t) - \sum_{t \in T} \sum_{i \in G} \sum_{b=1}^{N_{Gi}} \rho_{n(i)}(t) P_{Gib}(t). \qquad (4.144)$$

Finally, the declared social welfare is defined as the total profits of the buyers (the consumers) minus the total costs of the sellers (the generating companies), for the whole multi-period framework. The declared social welfare is computed through the expression below:

$$\mathrm{DSW} = \sum_{t \in T} \sum_{j \in D} \sum_{k=1}^{N_{Dj}} \lambda^{\mathrm{B}}_{Djk}(t) P_{Djk}(t) - \sum_{t \in T} \sum_{i \in G} \sum_{b=1}^{N_{Gi}} \lambda^{\mathrm{B}}_{Gib}(t) P_{Gib}(t). \qquad (4.145)$$

It should be noted that if the generating companies do not bid at their respective marginal costs and do not consider fixed, start-up and shut-down costs, the second term of the objective function is not actually the cost of the generating unit. And if the consumers do not bid at their respective marginal utilities, the first term is not the profit of the consumer. Therefore, the above expression represents the declared social welfare. The actual social welfare is computed as the sum of the producer surplus, the consumer surplus and the merchandising surplus:

$$\mathrm{SW} = \mathrm{PS} + \mathrm{CS} + \mathrm{MS}. \qquad (4.146)$$

4.6 Example

This example illustrates the procedure to clear a multi-period equilibrium market if the generating units impose a minimum profit requirements.

4.6.1 Data

Topology and line data can be found in Subsection 3.2.5.1. Generating unit and demand data are presented in Subsection 4.3.4.1. The generating units 1 and 2 declare a total minimum profit condition on the whole time horizon equal to 0 \$/h and 250 \$/h, respectively.

4.6.2 Multi-Period Equilibrium / Near-Equilibrium Including Minimum Profit Requirements

The multi-period equilibrium considering that the generating units 1 and 2 impose a minimum profit conditions of \$ 0 and \$ 250, respectively, is solved using Benders decomposition Algorithm 4.2 explained in Subsection 4.4.4. The solution has been achieved in 9 iterations and 1.5 seconds of CPU within a relative tolerance lower than 0.001. The computer used is a Dell PowerEdge 6600 with 4 processors at 1.60 GHz and 2 GB of RAM memory. Note that the value of Z_{QPP} in the solution is equal to zero; therefore, there are no complementarity infeasibilities.

Table 4.8 shows results concerning the power output, the revenue and the profit of each generating unit in each time period. The total profit of units 1 and 2 throughout the whole multi-period framework are \$ 777 and \$ 0, respectively. Start-up costs on the market time horizon are zero for both generating units. Note that generating unit 2 has been expelled from the market because the cost, for the system, of increasing prices to satisfy the minimum profit requirement of that unit is higher than the cost, for the system, of expelling it from the market and of satisfying all demand with generating unit 1. In the period with the highest demand, period 2, generating unit 1 is producing its maximum capacity.

Table 4.8: Results for the generating units. Example 4.6

Period	Unit	Power output [MW]	Revenue [$/h]	Profit [$/h]
1	1	148.48	3014.17	165.00
	2	0.00	0.00	0.00
2	1	200.00	4342.78	447.78
	2	0.00	0.00	0.00
3	1	184.29	3741.16	165.00
	2	0.00	0.00	0.00

Table 4.9 provides the consumed power and demand cost for each demand in each time period. Note that the highest power consumption in the system takes place in period 2.

Locational marginal prices at each node and for each time period are presented in Table 4.10. Note that prices are higher in the period with higher demand.

4.6.3 Comparison with no Minimum Profit Conditions Case

This subsection provides a comparison between the market solution obtained if minimum profit conditions are taken into account and the market solution without considering minimum profit requirements.

For the case that involves minimum profit conditions, locational marginal prices increase for all nodes in the three time periods (Figure 4.4). The reason

Table 4.9: Results for the demands. Example 4.6

Period	Node	Power consumed [MW]	Demand cost [$/h]
1	3	80.40	1653.63
	4	67.00	1378.02
2	3	120.00	2663.14
	4	77.92	1714.32
3	3	99.60	2066.48
	4	83.00	1707.10

Table 4.10: Locational marginal prices. Example 4.6

Node	Period 1 [$/MWh]	Period 2 [$/MWh]	Period 3 [$/MWh]
1	20.30	21.71	20.30
2	20.66	22.10	20.66
3	20.57	22.19	20.75
4	20.57	22.00	20.57

of this price increment is that unit 2 is expelled from the market for which makes the more expensive unit 1 increases its production.

Results for the generating units are illustrated in Figures 4.5 and 4.6. Figure 4.5 shows changes in power generated for each unit for the three time periods. Note that power generated for unit 2 is zero if minimum profit conditions are included because this unit has been expelled from the market. Therefore, power generated for the unit 1 increases in the three time periods to compensate this decrease in production. On the other hand, Figure 4.6 illustrates changes in profit for each unit for the three time periods. Note that profit for unit 1, if minimum profit conditions are considered, is higher in the three time periods while profit for unit 2 is zero.

Regarding results for demands, note in Figure 4.7 that demand in the three time periods decreases or remains equal if minimum profit conditions are included. The same behavior is observed in demand costs, Figure 4.8, except to the demand at node 4 in periods 1 and 3, whose cost increases due to these nodal prices increases.

Table 4.11 provides an economic comparison of the multi-period equilibrium with and without Minimum Profit Conditions (MPC) for the example. We observe that the consideration of minimum profit conditions implies an increase in the producer surplus and a decrease in the consumer surplus. If minimum profit conditions are considered, social welfare is lower than if it is

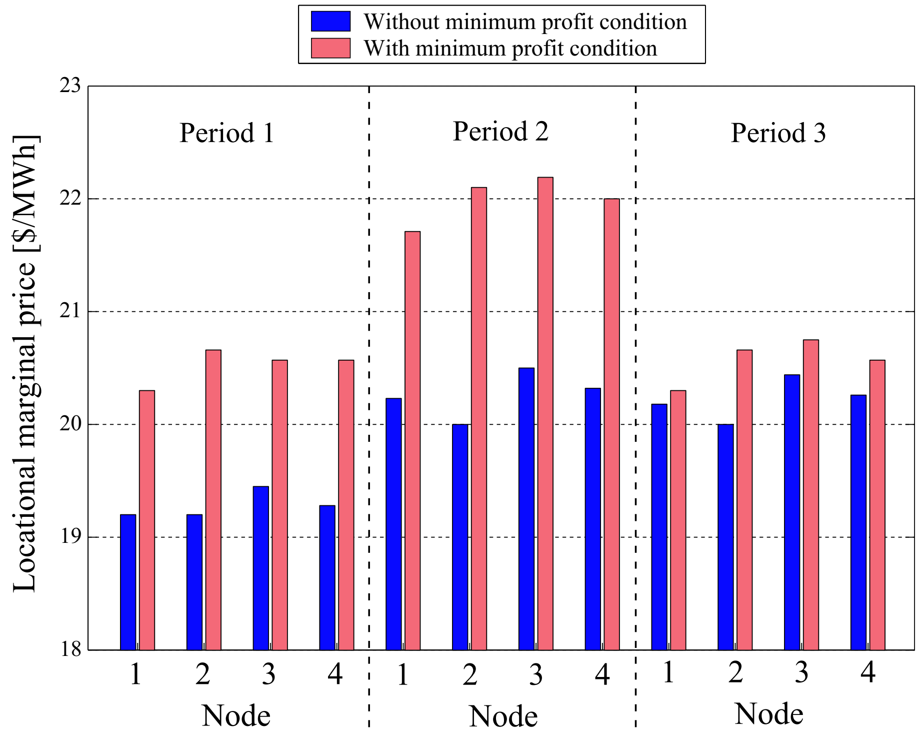

Figure 4.4: Comparison in terms of LMP. Example 4.6

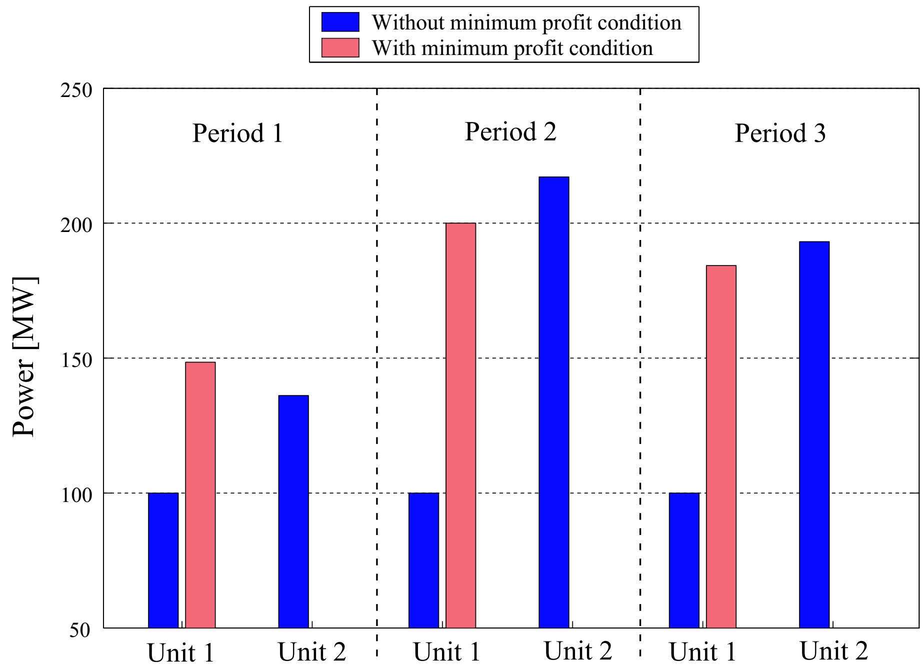

Figure 4.5: Comparison in terms of generating power. Example 4.6

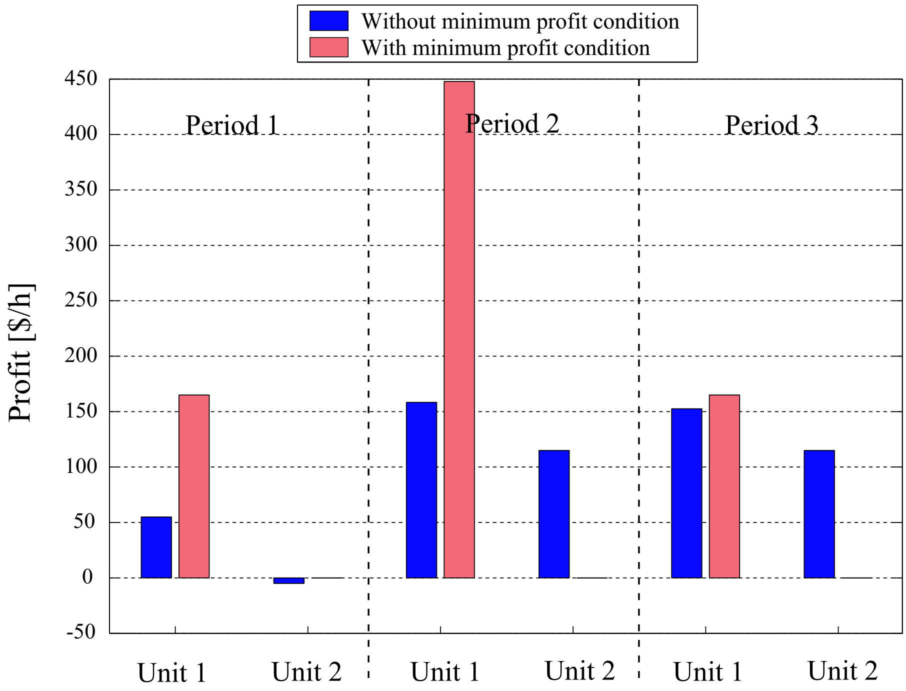

Figure 4.6: Comparison in terms of profit. Example 4.6

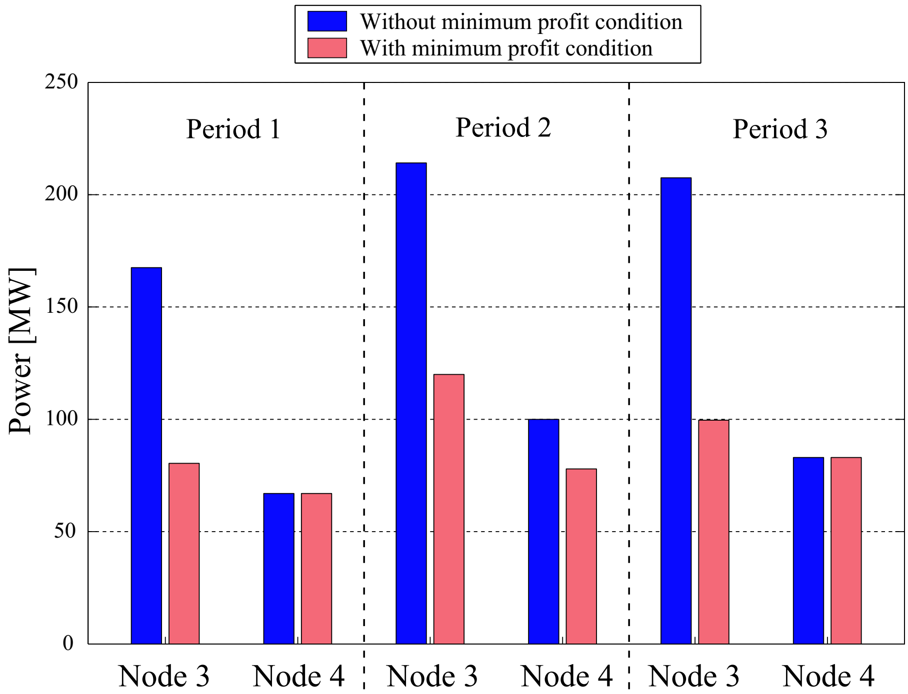

Figure 4.7: Comparison in terms of power consumed. Example 4.6

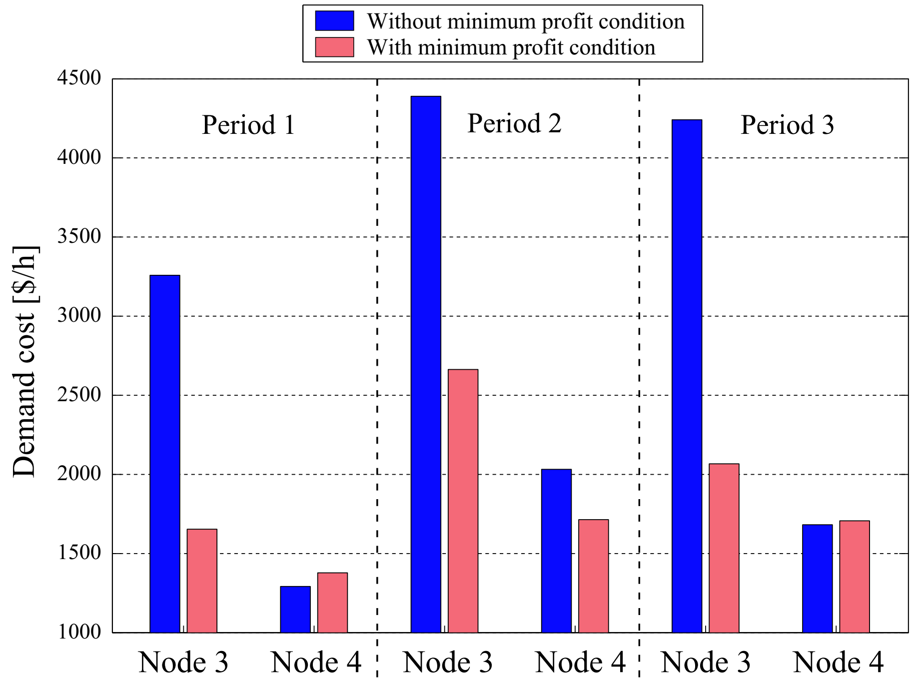

Figure 4.8: Comparison in terms of demand cost. Example 4.6

not. The reason for the significant decrease in the consumer surplus and the social welfare is that the power consumed in the system is significantly lower because generating unit 2 has been expelled from the market, if minimum profit conditions are considered.

Table 4.11: Comparison of economic metrics. Example 4.6

	Multi-period equilibrium without MPC	Multi-period equilibrium with MPC	Difference [%]
Producer surplus [$]	590.87	777.78	31.6
Consumer surplus [$]	1581.50	881.64	-44.3
Merchandising surplus [$]	115.2	84.57	-26.6
Social welfare [$]	2287.57	1743.99	-23.8
Declared social welfare [$]	2317.58	1758.99	-24.1

Finally, it should be noted that the large percentage changes that appear in Table 4.11 do not generally occur in realistic markets. For the sake of clarity, this example is actually designed so that large changes occur if minimum profit conditions are imposed.

4.7 Summary

This chapter models a multi-period equilibrium for an electricity market. The multi-period equilibrium is defined as the generating company / consumer energy transaction levels and their associated prices that result in maximum profit for every generating company, maximum utility for every consumer, and maximum social welfare for the whole multi-period framework, while inter-temporal constraints are enforced. First, we formulate the multi-period equilibrium without considering minimum profit conditions using Benders decomposition to avoid the limitations imposed by the necessary use of binary variables to model the on / off decisions. Benders decomposition technique decomposes the problem into a mixed-integer linear master problem to compute the optimal values for the binary variables, and a quadratic subproblem to obtain the optimal values for the continuous variables. Then, conditions for ensuring minimum profit levels for generating units are included in the equilibrium model. This relevant model is also formulated using Benders decomposition and minimum profit conditions are included as additional constraints of the subproblem. We propose three techniques to solve the resulting nonlinear subproblem.

Chapter 5

Case Studies

5.1 Introduction

In this chapter, the equilibrium procedures proposed in Chapters 3 and 4 are illustrated in case studies based on the IEEE 24-node Reliability Test System (RTS) [47].

First, in order to illustrate the impact of minimum profit constraints on the single-period equilibrium models explained in Chapter 3, a comparison of results from models with and without minimum profit constraints is provided.

Next, the multi-period equilibrium procedures presented in Chapter 4 are applied to several case studies to analyze the market behavior if both coupling constraints and minimum profit conditions are included.

5.2 Single-Period Case

In this section, the single-period market equilibrium solution technique is illustrated in several case studies based on the IEEE 24-node RTS. First, generating company data, consumer data and network data for the IEEE RTS are presented. After that, the single-period equilibrium is obtained for this system considering that no generating unit imposes minimum profit conditions. Then, the single-period equilibrium model is applied to the same system but imposing minimum profit conditions on some generating units. After that, a comparison of results from both cases is carried out. Finally, the behavior of the single-period equilibrium as minimum profit conditions are successively more restrictive is illustrated through several case studies.

5.2.1 Data

This subsection presents some observations on the IEEE 24-node RTS. A detailed account of all the data is provided in Appendix C.

The transmission network consists of 24 nodes connected by 38 lines and transformers. The transmission lines include two voltage levels, 138 kV and 230 kV. There are 32 generating units connected throughout the network, with two nuclear, six hydroelectric and the rest thermal. We suppose that the hydroelectric units have enough water in their respective reservoirs to produce their maximum capacity during each period of the considered time horizon; therefore, no additional constraints must be included in the problem of the generating company. The maximum generating capacity of the system (all the generating units) is 3405 MW. There is electricity demand in 17 nodes of the network. Although not relevant for this thesis, note that the system has voltage corrective devices at node 14 (synchronous condenser) and node 6 (reactor).

The generating unit data can be found in Appendix C. The capacity of each unit and the node which each unit is connected to are in Table C.1. Table C.2 provides operating cost data. The size (MW) and the price bids ($/MWh) of each unit are provided in Table C.4. In the case studies of this chapter, price bids and marginal costs are the same for reasons of simplicity. Moreover, we consider that every generating company only owns one generating unit, therefore there are 32 generating companies.

Demand data are also given in Appendix C. Note that loads are named according to their location in the network. Table C.5 provides demand bids for the peak load hour and Table C.6 includes minimum demand requirements for each demand. For simplicity, it is considered that price bids by each demand correspond to its marginal utilities, and that each consumer has a single demand, therefore there are 17 consumers.

Topology and line data including transmission capacity limits of the lines are presented in Appendix C, Figure C.1 and Table C.10. The transmission capacity limit of line 14-16 is reduced from 500 MW to 340 MW in the case studies of this section (single-period case) so that congestion occurs. The number of blocks used to linearize losses are eight.

5.2.2 No Minimum Profit Condition Case

A pool-based electricity market is considered and the single-period market equilibrium is obtained for the IEEE RTS if no minimum profit conditions are imposed by the generating units. This equilibrium is computed directly solving the mixed linear complementarity problem defined by equations (3.18)-(3.33), Subsection 3.2.2.

Table 5.1 provides equilibrium results concerning power outputs, revenues and profits for the pool-based electricity market if no Minimum Profit Conditions (MPC) are imposed by the generating units. It can be noted that all generating units are on-line except the most expensive ones, that is, 12 MW units (units 15-19) and 20 MW units (units 1, 2, 5 and 6). The power capacity of the nuclear units (units 22 and 23) is large and their operating costs are low which result in high profits for these types of units.

Table 5.1: Results for the generating units. Single-period equilibrium without MPC

Unit	Power output [MW]	Revenue [$/h]	Profit [$/h]
1, 2	0.00	0.00	0.00
3, 4	76.00	1572.18	565.86
5, 6	0.00	0.00	0.00
7, 8	76.00	1581.83	575.51
9-11	57.33	1242.40	117.75
12-14	118.20	2409.51	84.91
15-19	0.00	0.00	0.00
20	155.00	3056.42	1440.93
21	155.00	3055.33	1439.84
22	400.00	7690.53	5505.13
23	400.00	7658.53	5473.13
24-29	50.00	929.59	929.59
30, 31	155.00	3042.49	1427.01
32	350.00	6870.15	3123.57
Total	2900.60	57257.19	28304.86

Table 5.2 provides the power consumed and the corresponding demand costs. The total losses are the difference between the total generating power and the consuming power, and are equal to 50.51 MW.

The equilibrium of this pool-based electricity market provides Locational Marginal Prices (LMP) for all the nodes, which are provided in Table 5.3. It should be noted that price differences throughout the network are small as no significant congestion occurs in this network, but there is a particular behavior worth mentioning. Locational marginal prices in nodes 1-14 are higher than the locational marginal prices in nodes 15-24. This fact is due to a slight overloading in line 14-16 (power flow through line 14-16 is equal to the maximum capacity limit of this line) that splits the system into two areas, one with an excess of expensive generation, nodes 1-14, and another one with inexpensive generation, nodes 15-24.

A centralized optimal power flow is considered for the same system and solved through the linear programming problem defined by equations (3.34)-(3.46), Subsection 3.2.4. Results obtained are the same as those provided by the single-period equilibrium except for generating units 9, 10 and 11, and demand 7. Power output for these units are 50 MW, 71 MW and 50

Table 5.2: Results for the demands. Single-period equilibrium without MPC

Demand	Power consumed [MW]	Demand cost [$/h]	Demand	Power consumed [MW]	Demand cost [$/h]
1	111.60	2308.62	10	188.50	3946.62
2	93.68	1949.81	13	260.47	5309.75
3	186.00	3815.28	14	187.54	3845.85
4	71.54	1527.33	15	327.57	6459.29
5	68.64	1449.31	16	103.34	2037.02
6	131.48	2815.59	18	344.10	6615.78
7	121.84	2640.22	19	187.04	3702.59
8	165.30	3659.05	20	132.27	2607.36
9	169.18	3529.43	Total	2850.09	58218.88

Table 5.3: Locational marginal prices. Single-period equilibrium without MPC

Node	Locational marginal price [$/MWh]	Node	Locational marginal price [$/MWh]
1	20.69	13	20.39
2	20.81	14	20.51
3	20.51	15	19.72
4	21.35	16	19.71
5	21.11	17	19.31
6	21.41	18	19.23
7	21.67	19	19.80
8	22.14	20	19.71
9	20.86	21	19.15
10	20.94	22	18.59
11	20.77	23	19.63
12	20.73	24	20.31

MW, respectively. Revenues are 1083.50 $/h, 1539.57 $/h and 1083.50 $/h; and profits are 117.75 $/h for the three units. Concerning demand 7, the power consumed is 120.84 MW and the cost is 2618.60 $/h. The economic efficiency of the solution of the single-period equilibrium and of the optimal power flow solution are the same as the producer surplus, consumer surplus, merchandising surplus and social welfare are identical for both solutions, therefore, both of them have the same economic efficiency. Moreover, the problem has at least two solutions with identical objective function value.

5.2.3 Minimum Profit Condition Case

The following case study of the IEEE RTS is conducted to illustrate the proposed single-period market equilibrium under minimum profit conditions. Several generating units impose minimum profit conditions, as the required profit are amounts higher than their respective profits in equilibrium. We consider that the minimum profit requirements are 100 \$/h for units 12, 13 and 14. The resulting near-equilibrium is computed solving the quadratic programming problem defined by equations (3.63)-(3.93), Subsection 3.3.2.2. The solution of this problem has been obtained using the successive over-relaxation method (Algorithm 3.1 in Subsection 3.3.4.2). We have chosen the successive over-relaxation method to solve the problem because it converges faster than the other two algorithms reported in Subsection 3.3.4. The solution of the successive over-relaxation iterative method has been obtained in 4 iterations within a relative tolerance of 0.001, where $a = 1.1$ and $d = 0.8$.

Table 5.4 provides results concerning power output, revenues and profits under minimum profit conditions. Again, the more expensive units are off-line, and the rest are running at different load levels. Note that all minimum profit requirements imposed by the generating units are satisfied, and profit for units 12, 13 and 14 are equal to 99.92 \$/h (it is not exactly 100 \$/h due to numerical errors and the tolerance, but this result can be considered good enough), therefore no generating unit has been expelled from the market as a result of requiring a minimum profit condition.

Table 5.5 gives the power consumed and the corresponding demand costs. The losses are the difference between the power output in Table 5.4 and the total power consumed in Table 5.5, and are equal to 50.51 MW.

Table 5.6 provides locational marginal prices for the pool-based electricity market. The price differences throughout the network is due to a slight congestion that occurs in line 14-16, which causes prices in nodes 1-14 to be higher than in nodes 15-24.

Minimum profit conditions generate small complementarity infeasibilities since the objective function optimal value of problem (3.63)-(3.93) is slightly above zero, exactly 1.5 \$/h. This slight infeasibility is partly related to the fact that in node 13 of the network, the locational marginal price is higher than the marginal utility of demand 13. Therefore, equations (3.70) and (3.73) for node 13 are not complementary, that is, the complementarity condition of both equations is sligthly above zero. The power consumed by block 2 of the demand in node 13 is 4.29 MW, the marginal utility of this block is 20.38 \$/MWh and the price in this node is 20.51 \$/MWh. The

Table 5.4: Results for the generating units. Single-period equilibrium with MPC

Unit	Power output [MW]	Revenue [$/h]	Profit [$/h]
1, 2	0.00	0.00	0.00
3, 4	76.00	1572.27	565.95
5, 6	0.00	0.00	0.00
7, 8	76.00	1581.92	575.61
9	71.00	1538.57	117.75
10, 11	50.00	1083.50	117.75
12-14	118.20	2424.52	99.92
15-19	0.00	0.00	0.00
20	155.00	3056.60	1441.11
21	155.00	3061.28	1445.80
22	400.00	7704.68	5519.28
23	400.00	7672.63	5487.23
24-29	50.00	931.30	931.30
30, 31	155.00	3048.43	1432.94
32	350.00	6883.55	3136.97
Total	2899.60	57350.91	28420.20

Table 5.5: Results for the demands. Single-period equilibrium with MPC

Demand	Power consumed [MW]	Demand cost [$/h]	Demand	Power consumed [MW]	Demand cost [$/h]
1	111.60	2308.75	10	188.50	3954.32
2	93.68	1949.93	13	260.47	5342.84
3	186.00	3815.50	14	187.54	3853.35
4	71.54	1527.42	15	327.57	6459.68
5	68.64	1452.14	16	103.34	2040.99
6	131.48	2821.08	18	344.10	6627.95
7	120.84	2618.60	19	187.04	3709.81
8	165.30	3666.18	20	132.27	2612.45
9	169.18	3536.31	Total	2849.09	58297.30

Table 5.6: Locational marginal prices. Single-period equilibrium with MPC

Node	Locational marginal price [$/MWh]	Node	Locational marginal price [$/MWh]
1	20.69	13	20.51
2	20.81	14	20.55
3	20.51	15	19.72
4	21.35	16	19.75
5	21.16	17	19.34
6	21.46	18	19.26
7	21.67	19	19.83
8	22.18	20	19.75
9	20.90	21	19.18
10	20.98	22	18.63
11	20.81	23	19.67
12	20.77	24	20.31

complementarity condition of equations (3.70) and (3.73) is

$$0 \leq \rho_{13} - \lambda^{\mathrm{U}}_{D13,2} - \sigma_{13} + \varphi_{13,2} \ \perp \ P_{D13,2} \geq 0, \tag{5.1}$$

and replacing the corresponding values, we obtain,

$$0 \leq 20.51 - 20.38 - 0 + 0 = 0.13 \ \perp \ 4.29 \geq 0. \tag{5.2}$$

Any one of the above two conditions is not satisfied as equality and therefore there is no complementarity. The infeasibility caused by this lack of complementarity is,

$$0.13 \times 4.29 = 0.56 \text{ \$/h}. \tag{5.3}$$

As was mentioned before, this infeasibility is originated by the fact that the demand in node 13 must pay an energy price higher than its utility for a certain energy amount, so this demand has an economic loss. We can compensate the demand 13 for this economic loss paying it an uplift equal to its loss. This uplift could be allocated pro-rata among all the market participants. Demand 13 is paid an uplift equal to the cost incurred due to infeasibility, that is, $C = 0.56$ $/h. This cost is assigned half to the generating units and half to the demands, and each half cost is allocated pro-rata among generating units or among demands. Therefore, the additional cost of each generating unit, C_i, and of each demand, C_j, is computed using the following equations.

$$C_i = \frac{C}{2} \frac{\sum_{b=1}^{N_{Gi}} P_{Gib}}{\sum_{i=1}^{N_G} \sum_{b=1}^{N_{Gi}} P_{Gib}}; \ \forall i \in G \tag{5.4}$$

$$C_j = \frac{C}{2} \frac{\sum_{k=1}^{N_{Dj}} P_{Djk}}{\sum_{j=1}^{N_D} \sum_{k=1}^{N_{Dj}} P_{Djk}}; \ \forall j \in D; \forall j \neq 13 \tag{5.5}$$

$$C_j = \frac{C}{2} \frac{\sum_{k=1}^{N_{Dj}} P_{Djk}}{\sum_{j=1}^{N_D} \sum_{k=1}^{N_{Dj}} P_{Djk}} - C; \text{ for j} = 13. \tag{5.6}$$

Table 5.7 provides the cost assigned to each generating unit due to the infeasibility cost of the single-period near-equilibrium. Note that these costs are subtracted from the corresponding profit of each generating unit once the near-equilibrium has been found, and the resulting profit for each generating unit is shown in the last column of this table. In addition, Table 5.8 provides the cost assigned to each demand due to this infeasibility cost. The last column of the table provides the final demand cost of each demand once the corresponding infeasibility cost has been added. Note that the infeasibility cost assigned to each unit and demand are insignificant.

5.2.4 Comparison between Single-Period Equilibrium with and without Minimum Profit Conditions

A detailed comparison of results from single-period equilibria with and without minimum profit conditions (Subsections 5.2.2 and 5.2.3, respectively) is carried out in this Subsection.

Note that the power produced by the generating units and the power consumed by the demands are identical for both cases except for units 9, 10 and 11 and demand 7. Units 9, 10 and 11, located at node 7, modify their power outputs decreasing the total generation in node 7. This generating decrease causes a consumption decrease in the same node by demand 7. Therefore, these power changes do not affect the rest of the network.

Locational marginal prices increase to satisfy minimum profit conditions if these constraints are imposed. Table 5.9 provides locational marginal prices

Table 5.7: Cost of the generating units due to infeasibility. Single-period equilibrium with MPC

Unit	Infeasibility cost [$/h]	Profit [$/h]
1, 2	0.000	0.00
3, 4	0.007	565.94
5, 6	0.000	0.00
7, 8	0.007	575.60
9	0.007	117.74
10, 11	0.005	117.75
12-14	0.011	99.91
15-19	0.000	0.00
20	0.015	1441.10
21	0.015	1445.79
22	0.039	5519.24
23	0.039	5487.19
24-29	0.005	931.30
30, 31	0.015	1432.93
32	0.034	3136.93

throughout the network for the cases with and without minimum profit constraints, and the variation of both cases with respect to the case without minimum profit conditions. Note that changes in prices due to minimum profit conditions are comparatively larger at nodes where the generating units imposing minimum profit constraints are located (price variation around 0.6 %) than in the rest of the nodes (no price variations or price variations around 0.2 %). However, these constraints have an influence throughout the system as reflected in Table 5.9. Figure 5.1 shows locational marginal prices obtained as a result of the single-period equilibrium with and without minimum profit conditions.

Figure 5.2 compares generating profits for the equilibrium with and without minimum profit conditions. Note that only those generating units at nodes whose prices increase, experience increments in revenues and profits. For instance, generating unit 22, located at node 18, produces 400 MW in both cases (with and without minimum profit constraints), as stated in Tables 5.1 and 5.4. However, the profit of unit 22 is higher in the case with minimum profit conditions (5519.24 $/h) than in the case without these conditions (5505.13 $/h). This is due to a change in the LMP of node 18 that increases from 19.23 $/MWh to 19.26 $/MWh. Figure 5.2 shows that profit

Table 5.8: Cost of the consumers due to infeasibility. Single-period equilibrium with MPC

Demand	Infeasibility cost [$/h]	Demand cost [$/h]
1	0.011	2308.76
2	0.009	1949.94
3	0.018	3815.52
4	0.007	1527.43
5	0.007	1452.15
6	0.013	2821.10
7	0.012	2618.61
8	0.016	3666.20
9	0.017	3536.33
10	0.019	3954.34
13	-0.534	5342.31
14	0.018	3853.37
15	0.032	6459.71
16	0.010	2041.00
18	0.034	6627.98
19	0.018	3709.83
20	0.013	2612.41

increments for the units that have imposed minimum profit requirements (units 12, 13 and 14) are much higher (profit variation equal to 17.5 %) than for the rest of the units (profit variations lower than 0.5 %).

Figure 5.3 shows a comparison in terms of demand costs. Note that increments in demand costs are insignificant except in node 7, where cost decreases 0.8 % because the power demanded also decreases; and in node 13, where cost increases by 0.6 %.

Finally, an economic comparison is provided in Table 5.10 of the single-period equilibrium with and without minimum profit constraints. Note that the last column of the table represents the variation of both cases with respect to the case without minimum profit conditions. If the generating units are allowed to impose minimum profit requirements, the producer surplus increases as a consequence of the price increments produced to satisfy these requirements. Analogously, the consumer surplus decreases because demand costs are higher. The merchandising surplus is defined as the total demand costs minus the total production revenues. In this case study, the merchandising surplus decreases if minimum profit conditions are considered. Note

Table 5.9: Changes in LMP. Single-period equilibrium with and without MPC

Node	Without MPC [$/MWh]	With MPC [$/MWh]	Difference [%]
1	20.69	20.69	0.006
2	20.81	20.81	0.006
3	20.51	20.51	0.006
4	21.35	21.35	0.006
5	21.11	21.16	0.195
6	21.41	21.46	0.195
7	21.67	21.67	0.000
8	22.14	22.18	0.195
9	20.86	20.90	0.195
10	20.94	20.98	0.195
11	20.77	20.81	0.195
12	20.73	20.77	0.195
13	20.39	20.51	0.623
14	20.51	20.55	0.195
15	19.72	19.72	0.006
16	19.71	19.75	0.195
17	19.31	19.34	0.195
18	19.23	19.26	0.184
19	19.80	19.83	0.195
20	19.71	19.75	0.195
21	19.15	19.18	0.184
22	18.59	18.63	0.184
23	19.63	19.67	0.195
24	20.31	20.31	0.006

that social welfare is the same in both cases. In Table 5.10, we also provide the total power produced and consumed in the system for both cases, with and without minimum profit conditions, and whose variation is insignificant.

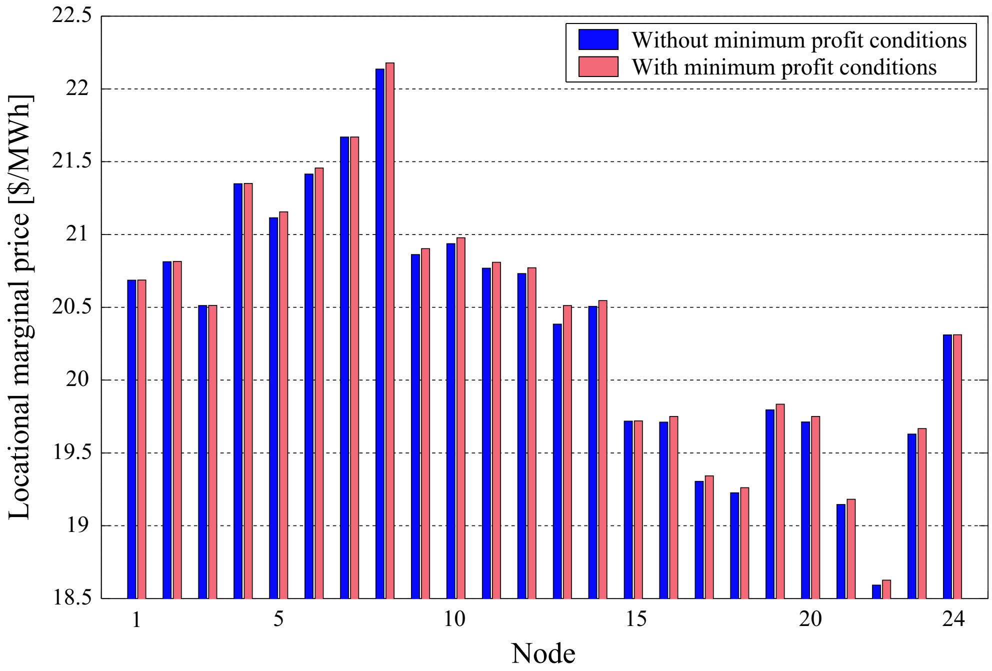

Figure 5.1: Comparison in terms of LMP. Single-period equilibrium with and without MPC

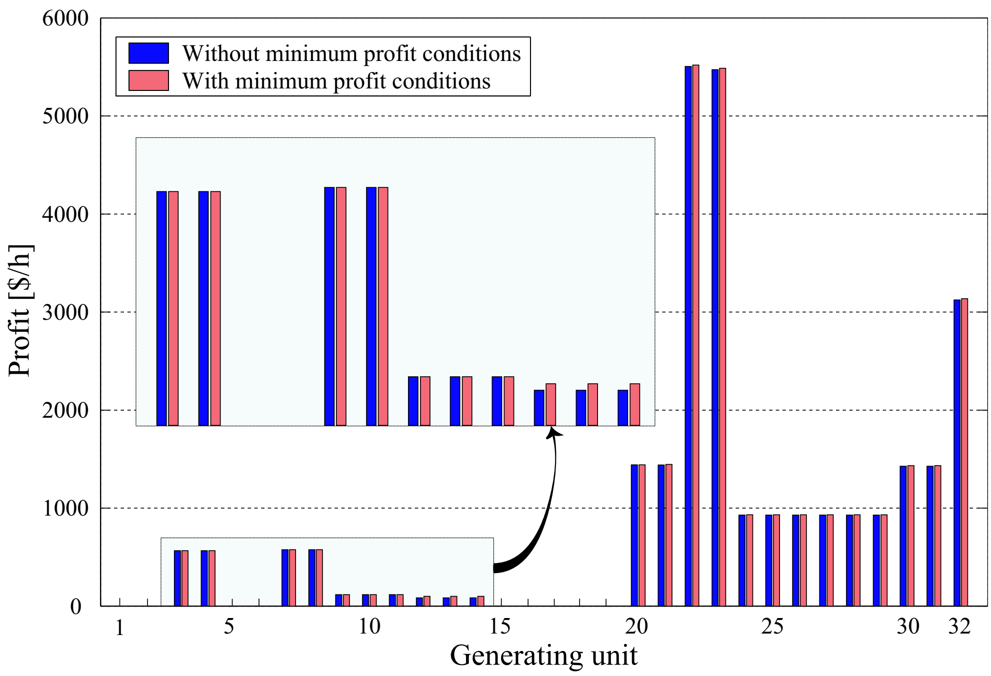

Figure 5.2: Comparison in terms of profit. Single-period equilibrium with and without MPC

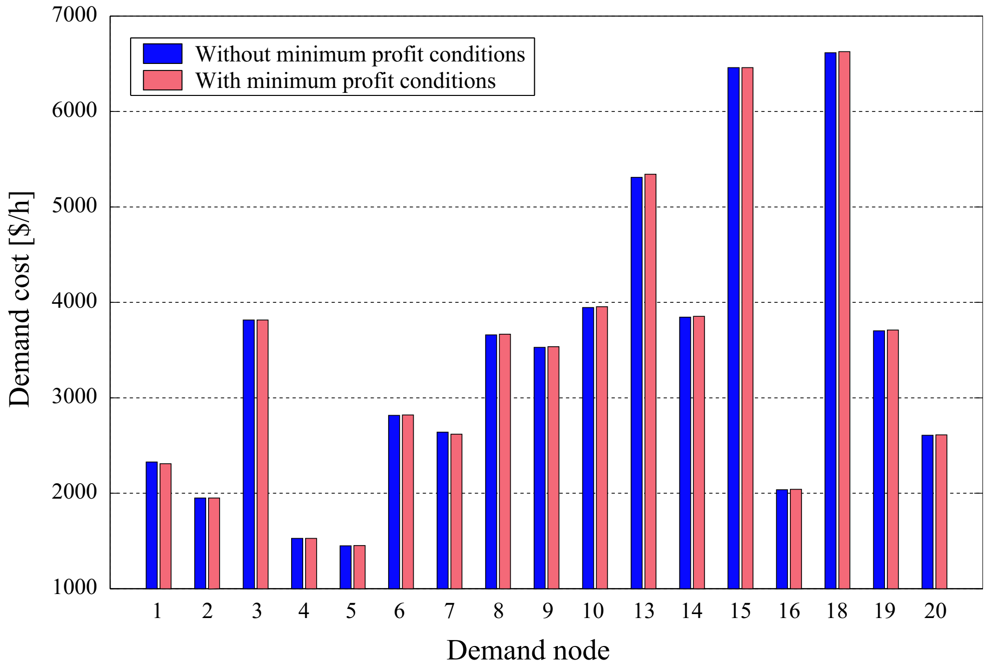

Figure 5.3: Comparison in terms of demand cost. Single-period equilibrium with and without MPC

Table 5.10: Comparison of relevant metrics. Single-period equilibrium with and without MPC

	Equilibrium without MPC	Equilibrium with MPC	Difference [%]
Power produced [MW]	2900.60	2899.60	-0.03
Power consumed [MW]	2850.09	2849.09	-0.04
Producer revenues [$/h]	57257.19	57350.91	0.16
Demand costs [$/h]	58218.88	58297.02	0.13
Producer surplus [$/h]	28304.86	28419.92	0.41
Consumer surplus [$/h]	5556.47	5456.72	-1.80
Merchandising surplus [$/h]	961.69	946.39	-1.59
Social welfare [$/h]	34823.03	34823.03	0.00

5.2.5 Evolution of the Market Equilibrium as Minimum Profit Conditions Change

To illustrate the effect of minimum profit conditions in the equilibrium, the single-period equilibrium has been derived as the generating units impose more and more restrictive minimum profit conditions. Three different cases are studied in detail. For case 1, generating units 12, 13 and 14 impose a minimum profit requirement of 100 $/h (Subsection 5.2.3), for case 2, 200 $/h, and for case 3, generating units 12 and 13 impose a 300 $/h profit requirement and generating unit 14 imposes 350 $/h. Results concerning the generating units, the demands and locational marginal prices for cases 2 and 3 are collected in Appendix D, Subsections D.1.1 and D.1.2, respectively.

As larger minimum profit constraints are imposed by the units, both generation and demand may change with respect to the case of no minimum profit constraints and any of the generating units imposing minimum profit may be expelled from the market, as can be seen in case 3.

In case 2, the power generated for the units remains constant with respect to the results of case 1, except for unit 9, which increases its generating power from 71.00 MWh to 79.33 MWh. This production increment involves an increment in the power consumed in node 7.

It should be noted that generating unit 14 is expelled from the market in case 3 due to the overly restrictive minimum profit constraint imposed by this unit in case 3. Note that the minimum profit imposed for unit 14 is about 250 % higher than its initial profit, 84.91 $/h. Generating unit 9 also changes its production to 75.63 MWh in this case. These changes in the generating power of the system cause a modification to the power consumed by the demands.

Figure 5.4 shows the evolution of locational marginal prices as minimum profit conditions become more stringent. Note that price differences throughout the network increase as higher profits are imposed by the generating units except at node 7 for case 2, where the locational marginal price does not change. Above in this figure we provide the percentage difference between LMP in cases 2 and 1 with respect to case 1 for each node, in the first row; and the percentage difference between LMP in cases 3 and 2 with respect case 2 for each node, in the second row.

Figure 5.5 represents generating profits for the three cases. Note that profits increase except for units 9, 10 and 11 in case 2. In case 3, profit for unit 14 is equal to zero because this unit has been expelled from the market.

Figure 5.6 compares demand costs for the three cases that consider different minimum profit requirements. Demand costs increase as minimum profit requirements are more restrictive. However, in case 3 demand costs decrease for some demands. This is because some demands change their consumption as a consequence of unit 14 being expelled from the market in case 3.

Table 5.11 provides an economic comparison between the three cases. In the last two columns, we find the percentage difference between case 2 and case 1 with respect to case 1, and between case 3 and case 2 with respect to case 2. As minimum profit conditions become more restrictive, the producer surplus increases due to price increments, and the consumer surplus decreases because demand costs are higher. The merchandising surplus increases as minimum profit conditions become more restrictive because power changes

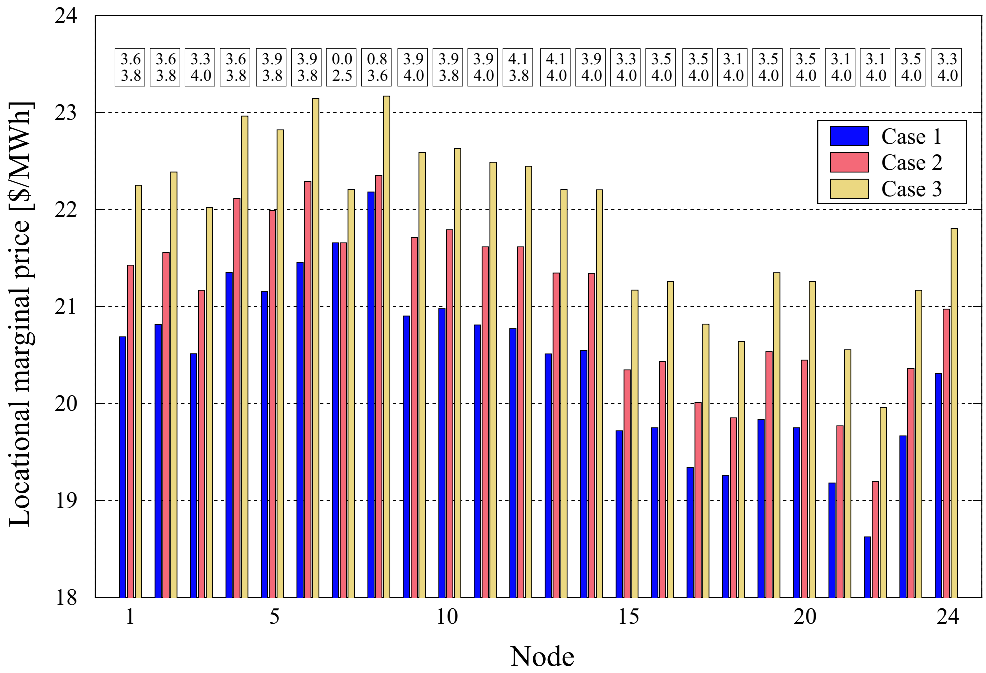

Figure 5.4: Comparison in terms of profit. Evolution of the single-period equilibrium

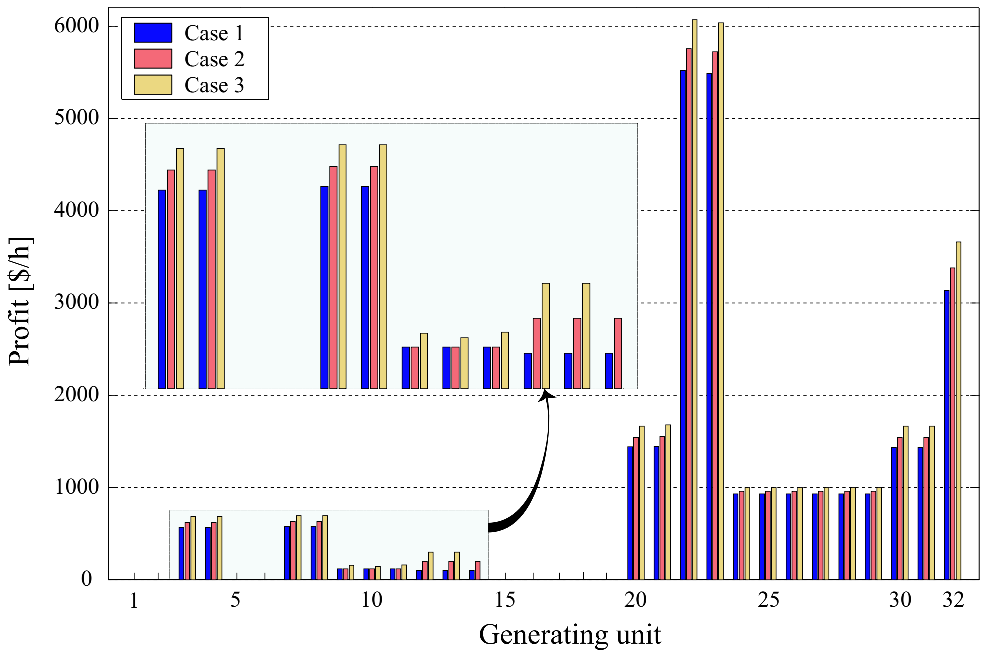

Figure 5.5: Comparison in terms of demand cost. Evolution of the single-period equilibrium

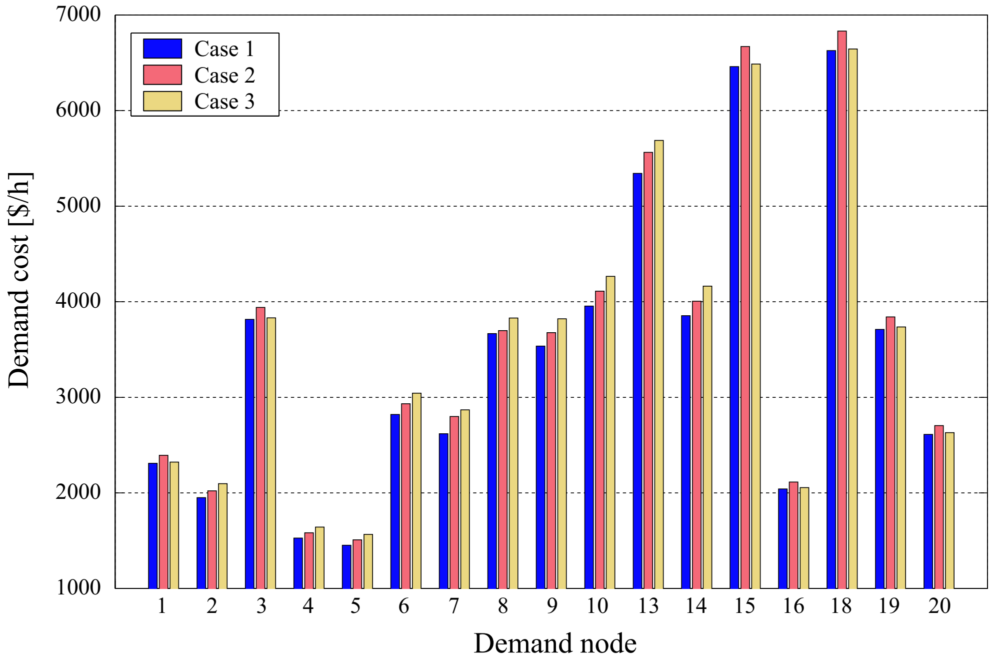

Figure 5.6: Comparison in terms of LMP. Evolution of the single-period equilibrium

originate congestion in the network. Social welfare remains unchanged in cases 1 and 2, but decreases in case 3. Note that both power produced and consumed decrease in case 3 because generating unit 14 is expelled from the market.

Table 5.11: Comparison of relevant metrics. Evolution of the single-period equilibrium

	Case 1	Case 2	Case 3	2 versus 1 [%]	3 versus 2 [%]
Power produced [MW]	2899.60	2907.93	2816.03	0.29	-3.16
Power consumed [MW]	2849.09	2857.42	2763.35	0.29	-3.29
Producer revenues [$/h]	57350.91	59370.15	59611.50	3.52	0.41
Demand costs [$/h]	58297.30	60357.20	60689.68	3.53	0.55
Producer surplus [$/h]	28420.20	30228.23	32252.68	6.36	6.70
Consumer surplus [$/h]	5456.44	3577.06	1376.55	-34.44	-61.52
Merchandising surplus [$/h]	946.39	1017.74	1080.43	7.54	6.16
Social welfare [$/h]	34823.03	34823.03	34709.66	0.00	-0.33

5.2.6 Problem Size

Table 5.12 provides the size of the mixed linear complementarity problem solved in order to obtain the single-period equilibrium without minimum profit conditions (column 1). This table also provides the size of the mixed-integer quadratic programming problem solved in each iteration of the successive over-relaxation iterative method to obtain the single-period equilibrium including minimum profit requirements of the generating units.

Table 5.12: Size of problems for single-period equilibrium models

Single-period without MPC		Single-period including MPC	
Number of positive continuous variables	2879	Number of continuous variables	3225
Number of free continuous variables	346	Number of binary variables	3
Number of equations	8887	Number of constraints	6078

Equilibrium results shown in this section have been obtained by solving their corresponding problems using the commercial solvers GAMS / PATH 4.6 (mixed linear complementarity problems) and GAMS / SBB with GAMS / MINOS 5.51 (mixed-integer quadratic programming problem). The relative tolerance of the successive over-relaxation iterative method for each case is 0.001. The computer used is a Dell PowerEdge 6600 with 4 processors at 1.60 GHz and 2 GB of RAM memory. Table 5.13 provides the CPU time required to solve the single-period equilibrium problems. The second row refers to the single-period equilibrium without minimum profit constraint (No MPC). If the single-period equilibrium for cases 1-3 is obtained using the linear approximation method explained in Subsection 3.3.4.1, the required CPU time is approximately ten times longer than values provided in the table. This table also provides the infeasibility cost for each case. This cost is incurred because some demands paying an energy price higher than their respective utilities; therefore, those demands are compensated with uplifts and the uplifts are allocated pro-rata among all the market participants. Note that the infeasibility cost is higher as minimum profit requirements are more restrictive, except to case 3. This decrease in the infeasibility cost could have been caused by the fact that generating unit 14 was expelled from the market provoking changes in the power produced and consumed. The last column of the table represents the percentage of the infeasibility cost with respect to the respective demand total cost. Note that those values are minor.

Table 5.13: CPU time and infeasibility cost for single-period equilibrium models

	CPU time [seconds]	Infeasibility cost [$/h]	Percentage [%]
No MPC	0.54	0.0	0.000
Case 1	4.82	0.5	0.000
Case 2	6.27	61.4	0.101
Case 3	17.01	4.5	0.007

5.3 Multi-Period Case

In this section, we discuss the differences between a multi-period equilibrium without considering minimum profit conditions and a succession of single-period equilibria using a case study based on the IEEE 24-node RTS. This comparison reveals the effect of inter-temporal constraints. Moreover, we illustrate the effect of imposing minimum profit conditions on the multi-period equilibrium.

This section is organized as follows. First, data of this case study are provided. Next, the multi-period equilibrium is obtained considering that no generating unit can impose minimum profit conditions. We compare this multi-period equilibrium with that obtained as a succession of single-period equilibria. Afterwards, the multi-period equilibrium is obtained if some generating units impose minimum profit requirements. Results are contrasted with the no minimum profit constrained case. Finally, we study the behavior of the multi-period equilibrium if minimum profit conditions are successively more restrictive. The size of the problems and the time burden involved in solving these problems are specified.

5.3.1 Data

The multi-period equilibrium is illustrated using a case study based on the IEEE 24-node RTS. We consider a time framework of 24 hours.

The generating unit data can be found in Appendix C. The locations of the generating units throughout the network are indicated in Table C.1. Table C.2 provides the operational cost for each unit type. In Table C.3, we find ramp rate limits, start-up and fixed costs. Note that we consider that ramp rate values of this table correspond to the ramp-up and ramp-down limits, and also to start-up and shut-down ramp limits. Finally, Table C.4 gives generating unit bids. These bids are assumed to be identical for all hours.

Demand data are also given in Appendix C. Tables C.5 and C.6 provide demand bids and the minimum demand requirements for the peak load hour.

For the sake of simplicity, price bids are considered constant throughout the 24 hours of the day. For this case study, hourly loads correspond to the Wednesday of week 51, therefore size bids and minimum demand values are different for every hour.

Topology and line data are presented in Subsection C.4. The transmission capacity limit of line 14-16 is reduced from 500 MW to 380 MW in the case studies of this section so that congestion occurs. The number of blocks used to linearize losses is four.

5.3.2 No Minimum Profit Condition Case

The multi-period equilibrium of the IEEE RTS is first obtained considering that the generating units cannot impose minimum profit conditions. This multi-period equilibrium is formulated in Subsection 4.3.1 and is solved through Algorithm 4.1 described in Subsection 4.3.2. The solution of the multi-period equilibrium has been achieved in 30 iterations within a relative tolerance lower than 0.01. The convergence behavior of Benders decomposition algorithm is illustrated in Figure 5.7. Observe the appropriate convergence of the algorithm and the smooth and monotonic increase of the lower bound of the optimal objective function value. Nevertheless, the lower bound progresses slowly once a reasonably small gap between the bounds is achieved. This is the typical behavior of Benders decomposition for large-scale problems.

Table 5.14 provides results for the multi-period equilibrium concerning generating unit production and profits on the whole time horizon for each generating unit. Note that the sum of the profits for the 24 hours is positive for every generating unit, although there are some generating units that has a negative profit for a particular hour. The generating units that remain off-line during the 24 hours correspond to the more expensive ones.

Table 5.15 provides results for the multi-period equilibrium with no minimum profit conditions concerning generating unit profits and revenues, demand costs, and minimum and maximum locational marginal prices for each time period. Observe that locational marginal prices are different at different nodes due to congestion during hours 1-24 in line 14-16 (at 380 MW), which splits the system into two areas, one with an excess of inexpensive generation and another one with expensive generation. There is a low and approximately constant demand during hours 1-6 that causes low locational marginal prices throughout the system because the on-line generating units are the more inexpensive ones. In hours 7, 8 and 9, demand increases sharply forcing more expensive generating units to start up, increasing locational marginal prices in the system. In hours 10-21, demand is high and approximately constant, and extra units with high costs start up to supply the demand. Finally, demand in hours 22-24 decreases sharply, forcing the more expensive generating units to shut down and / or to decrease the production, producing a decrease

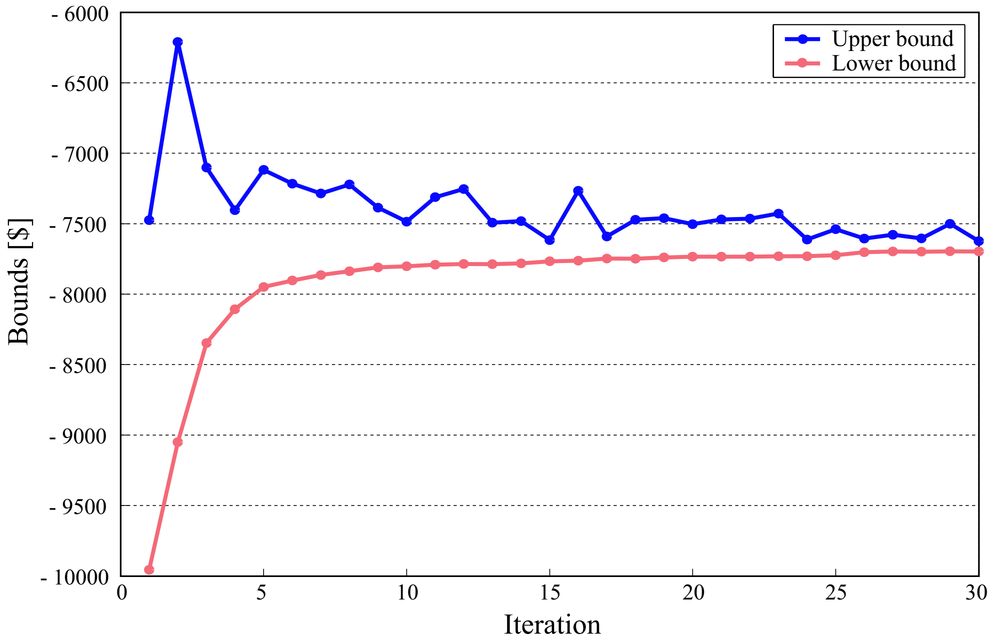

Figure 5.7: Evolution of bounds of the Benders decomposition. Multi-period equilibrium without MPC

Table 5.14: Results for the generating units. Multi-period equilibrium without MPC

Unit	Total energy [MWh]	Profit [k$]	Unit	Total energy [MWh]	Profit [k$]
1, 2	0.00	0.00	14	1700.29	0.74
3	1379.64	9.27	15-19	0.00	0.00
4	1314.80	8.86	20	3385.76	23.31
5, 6	0.00	0.00	21	3472.54	23.66
7	1395.81	8.89	22	9600.00	104.01
8	1466.69	9.34	23	9600.00	102.54
9	691.10	0.53	24-29	1200.00	18.73
10	1111.80	0.17	30, 31	3565.00	23.62
11	1094.43	1.13	32	7699.60	51.09
12, 13	0.00	0.00	Total	57042.46	484.43

in the locational marginal prices of the system. Note that unit profits are higher in hours with high demand. The same behavior is observed in unit revenues and demand costs.

Locational marginal prices for each node by time period, and power out-

Table 5.15: Results by time periods. Multi-period equilibrium without MPC

Hour	Gen. unit profit [k$/h]	Gen. unit revenue [k$/h]	Demand cost [k$/h]	Minimum LMP [$/MWh]	Maximum LMP [$/MWh]
1	7.20	23.59	24.33	10.41	13.43
2	7.44	21.52	22.25	10.14	13.08
3	6.92	19.95	20.67	9.88	12.73
4	6.88	19.57	20.17	9.86	12.57
5	6.88	19.57	20.17	9.86	12.57
6	6.92	19.95	20.67	9.88	12.73
7	10.03	29.60	30.43	11.79	15.20
8	27.05	48.65	49.70	18.02	22.11
9	27.15	52.71	53.64	18.44	22.17
10	28.29	52.89	53.91	18.47	22.17
11	28.29	52.89	53.91	18.47	22.17
12	28.17	52.71	53.64	18.44	22.17
13	28.17	52.71	53.64	18.44	22.17
14	28.17	52.71	53.64	18.44	22.17
15	27.59	52.01	52.95	18.22	22.17
16	27.74	52.21	53.15	18.27	22.17
17	29.69	55.85	56.71	19.01	22.17
18	29.70	56.42	57.28	19.01	22.17
19	32.24	58.87	59.77	19.94	23.24
20	29.28	53.67	54.76	18.82	23.07
21	27.18	51.30	52.34	18.09	22.17
22	23.24	43.87	44.85	16.59	20.92
23	20.54	38.16	39.49	15.68	20.45
24	8.41	22.40	23.45	10.53	14.37
Total	503.17	1003.78	1025.52	374.70	460.34

put and the profit of each generating unit by time period are collected in Appendix D, Subsection D.2.1.

Unlike a succession of single-period equilibria, a multi-period equilibrium approach takes inter-temporal coupling conditions properly into account. It is relevant to analyze the results obtained in both cases, so we have cleared the market using a succession of single-period equilibria. This succession of single-period equilibria is obtained solving a single-period equilibrium problem as the one stated in Chapter 3, Subsection 3.2.1, for each period, but including ramp rate constraints that only depend on the power production of the previous hour, i.e. ramp rate constraints are treated in a "greedy" fashion. Some results of the succession of single-period equilibria are reported in Appendix D, Subsection D.2.2.

Table 5.16 provides a comparison of the multi-period equilibrium without minimum profit conditions and its corresponding sequence of single-period equilibria. The last column presents the variation of both cases with respect to the multi-period equilibrium without minimum profit conditions. The very fact that multi-period equilibrium considers on / off status changes on the time horizon as optimization variables causes start-up costs to be lower in the multi-period equilibrium case. It can be observed that actual social welfare is 1% lower in the case of a sequence of single-period equilibria than in the multi-period equilibrium case, and power traded in the market is lower in the multi-period case. This fact implies that the multi-period equilibrium problem provides more efficient results than a succession of single-period equilibria, in addition to providing more realistic results. As compared with the more realistic multi-period equilibrium, a sequence of single-period equilibria overestimates the consumer surplus while it underestimates the producer surplus.

Table 5.16: Comparison of relevant metrics. Multi-period equilibrium versus the corresponding sequence of single-period equilibria

	Multi-period equilibrium	Sequence of single-period equilibria	Difference [%]
Total energy produced [MWh]	58242.44	59177.48	1.61
Total energy consumed [MWh]	56566.85	57792.85	2.17
Producer revenues [k$]	1003.81	1013.43	0.96
Demand costs [k$]	1025.51	1033.98	0.83
Start-up costs [k$]	3.06	5.09	66.34
Producer surplus [k$]	503.20	479.94	-4.62
Consumer surplus [k$]	237.44	253.93	6.94
Merchandising surplus [k$]	21.70	20.56	-5.25
Social welfare [k$]	762.34	754.42	-1.04
Declared social welfare [k$]	767.79	762.37	-0.71

Figure 5.8 illustrates the most important economic metrics for both multi-period without minimum profit conditions and a succession of single-period equilibria. Note that we represent the producer surplus, the consumer surplus, the merchandising surplus, the actual social welfare which corresponds to the sum of the previous metrics, and finally, the declared social welfare. We observe that the producer surplus is higher for the multi-period equilibrium case and the consumer surplus is lower for this case, while the merchandising

surplus is similar for both cases. We can see an increment in the actual social welfare for the multi-period equilibrium case. Differences in the declared social welfare are minor.

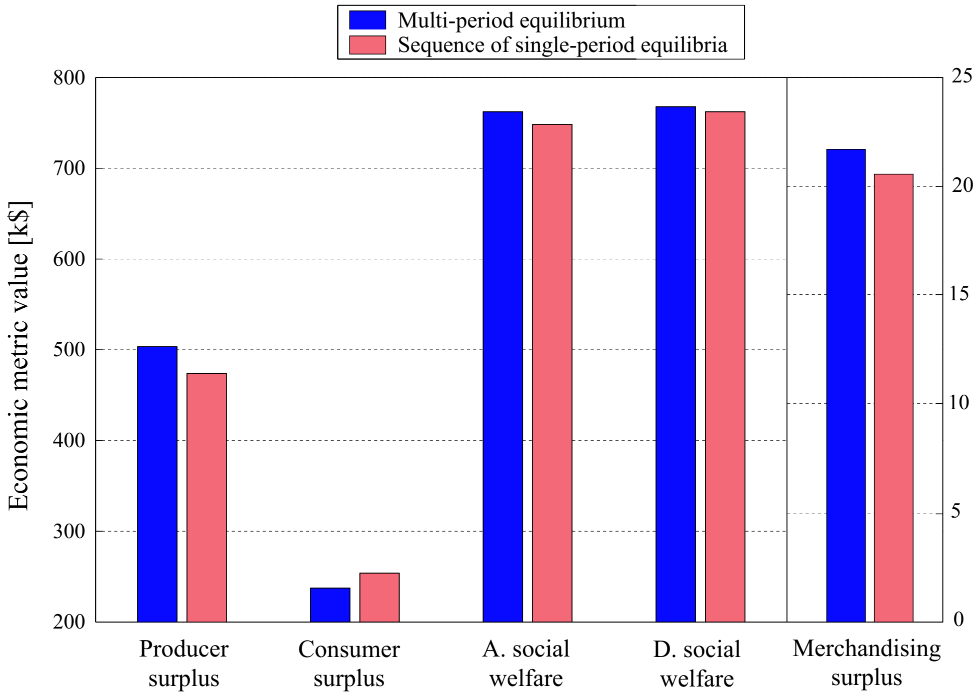

Figure 5.8: Economic efficiency metrics. Multi-period equilibrium versus the corresponding succession of single-period equilibria

5.3.3 Minimum Profit Condition Case

We obtain the multi-period equilibrium of the IEEE RTS considering that each of generating units 3 and 4 imposes a minimum profit requirement of \$ 9000 and generating units 9, 10 and 11 impose requirements of \$ 200. The rest of the generating units in the system impose a minimum profit requirement of \$ 0. This equilibrium is formulated in Subsection 4.4.2 and is obtained using Algorithm 4.2 in Subsection 4.4.4. The solution has been achieved in 18 iterations within a relative tolerance lower than 0.01. Note that there is no infeasibility on this solution. The convergence behavior of Benders decomposition is illustrated in Figure 5.9. As in Subsection 5.3.2, the no minimum profit condition case, we can observe a smooth increase of the lower bound of the optimal objective function value.

Table 5.17 provides results for the multi-period equilibrium problem concerning generating unit production and profits for the whole time horizon for two cases. In the first case, no generating unit is allowed to impose minimum

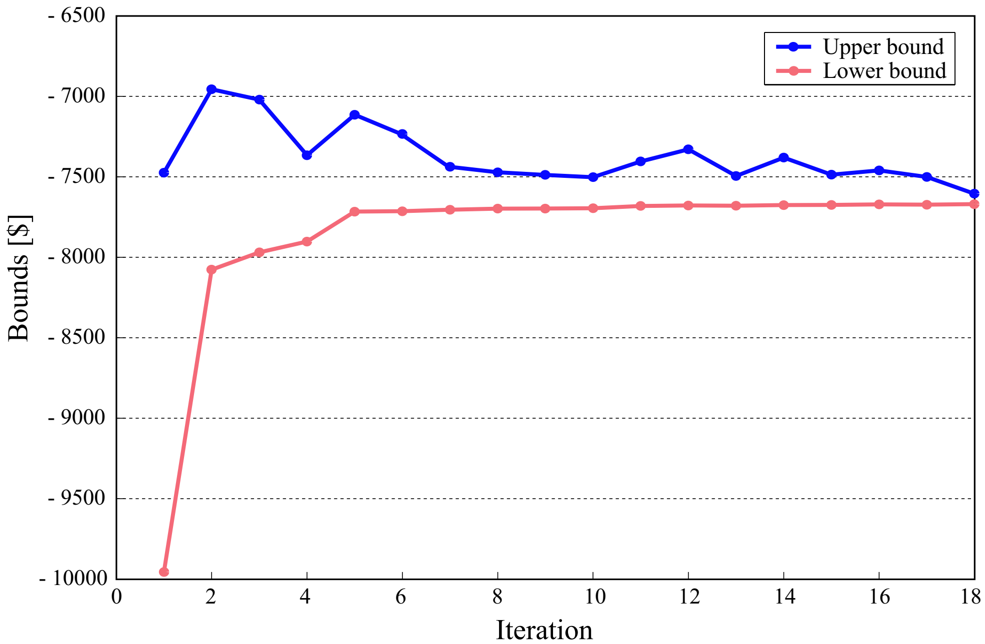

Figure 5.9: Evolution of bounds of the Benders decomposition. Multi-period equilibrium with MPC

profit conditions, corresponding to columns 3 and 4 in the table. These results correspond to those provided in Table 5.14 and are replicated here to facilitate the comparison. In the second case, the generating units impose the minimum profit conditions specified above, corresponding to columns 5 and 6 of the table. Note that in the first case, the profit for unit 4 is lower than \$ 9000, and the profit for unit 10 is lower than \$ 200. In the second case, we force that if these generating units are running in any time period of the market horizon, they must have profits at least equal to the minimum value they have imposed, that is, \$ 9000 and \$ 200, respectively. Also, note that in this second case, generating energy is redistributed so that the minimum profit requirements are satisfied. In fact, generating unit 4 increases its production and as a consequence its profit increases. Generating unit 10 decreases its production but changes the time periods when it is producing, therefore, this unit now produces power in hours with higher prices, as can be seen in the results shown in Appendix D.

Table 5.18 provides results for the multi-period equilibrium with minimum profit conditions concerning generating unit profits and revenues, demand load costs, and minimum and maximum locational marginal prices for each time period. These results are discussed below. We can observe the same evolution of prices as in Table 5.15 (without minimum profit conditions). Locational marginal prices are lower if load demands are low, that is, in hours 1-6. In hours 7-9 and 22-24 demand increases and decreases sharply,

Table 5.17: Results for the generating units. Multi-period equilibrium with MPC

Unit	Minimum profit requirement [k$]	Multi-period equilibrium without MPC		Multi-period equilibrium with MPC	
		Total energy [MWh]	Profit [k$]	Total energy [MWh]	Profit [k$]
1, 2	0.0	0.00	0.00	0.00	0.00
3	9.0	1379.64	9.27	1511.14	9.67
4	9.0	1314.80	8.86	1478.80	9.67
5, 6	0.0	0.00	0.00	0.00	0.00
7	0.0	1395.81	8.89	1470.90	9.72
8	0.0	1466.69	9.34	1530.20	9.72
9	0.2	691.10	0.53	0.00	0.00
10	0.2	1111.80	0.17	1040.00	0.98
11	0.2	1094.43	1.13	1271.47	1.86
12, 13	0.0	0.00	0.00	0.00	0.00
14	0.0	1700.29	0.74	1955.29	1.49
15-19	0.0	0.00	0.00	0.00	0.00
20	0.0	3385.76	23.31	3513.17	24.13
21	0.0	3472.54	23.66	3534.00	24.50
22	0.0	9600.00	104.01	9600.00	106.15
23	0.0	9600.00	102.54	9600.00	104.66
24-29	0.0	1200.00	18.73	1200.00	18.99
30	0.0	3565.00	23.62	2883.00	22.59
31	0.0	3565.00	23.62	3541.19	24.58
32	0.0	7699.60	51.09	7880.12	53.20
Total	—	57042.46	484.43	56809.28	497.87

respectively, increasing and decreasing locational marginal prices in the market, respectively. If the demand is high, hours 10-21, locational marginal prices are high. Again, unit profits are higher in hours with high demand. The same behavior is observed for unit revenues and demand costs.

Locational marginal prices for each node by time period, and power output and profit of each generating unit by time period are collected in Appendix D, Subsection D.2.3.

Table 5.18: Results by time periods. Multi-period equilibrium with MPC

Hour	Gen. unit profit [k$/h]	Gen. unit revenue [k$/h]	Demand cost [k$/h]	Minimum LMP [$/MWh]	Maximum LMP [$/MWh]
1	8.17	23.87	24.74	10.53	14.20
2	7.61	21.55	22.40	10.15	13.70
3	7.45	20.44	21.17	10.06	13.13
4	7.39	20.03	20.76	10.04	13.09
5	7.39	20.03	20.76	10.04	13.09
6	7.45	20.44	21.17	10.06	13.13
7	22.25	39.31	40.62	16.59	22.28
8	24.94	48.78	49.84	17.97	22.65
9	29.61	53.54	54.64	18.97	23.84
10	31.61	56.04	57.14	19.68	24.67
11	28.70	53.67	54.76	18.82	23.07
12	28.17	52.44	53.50	18.44	22.60
13	28.17	52.44	53.50	18.44	22.60
14	28.17	52.44	53.50	18.44	22.60
15	27.49	51.76	52.71	18.22	22.22
16	28.04	52.31	53.26	18.41	22.45
17	31.81	57.94	58.85	19.78	23.24
18	32.24	58.87	59.77	19.94	23.24
19	32.24	58.87	59.77	19.94	23.24
20	29.28	53.67	54.76	18.82	23.07
21	27.18	51.30	52.34	18.09	22.17
22	23.24	43.87	44.85	16.59	20.92
23	10.98	28.90	29.88	11.66	15.20
24	7.29	21.52	22.25	10.14	13.08
Total	516.87	1014.03	1036.94	379.82	473.48

5.3.4 Comparison between Multi-Period Equilibrium with and without Minimum Profit Conditions

This subsection presents a comparison of results of the multi-period equilibrium without minimum profit conditions, Subsection 5.3.2, and results of the multi-period equilibrium with minimum profit conditions, Subsection 5.3.3.

Figure 5.10 shows a variation range of locational marginal prices for each hour for both cases, with and without minimum profit constraints. The maximum and minimum locational marginal price for each hour of each case is represented by a small square. The evolution of locational marginal prices on the time horizon can be clearly observed. We can see that maximum price values for the minimum profit constrained case are generally higher than for the other case, except for the last two hours, which involve lower prices. In hours 7, 9 and 10 substantial maximum price differences are observed between the two cases, due to the fact that the off-line unit with lower operating costs, that is, generating unit 9, is not started up because minimum profit requirements for this unit cannot be satisfied.

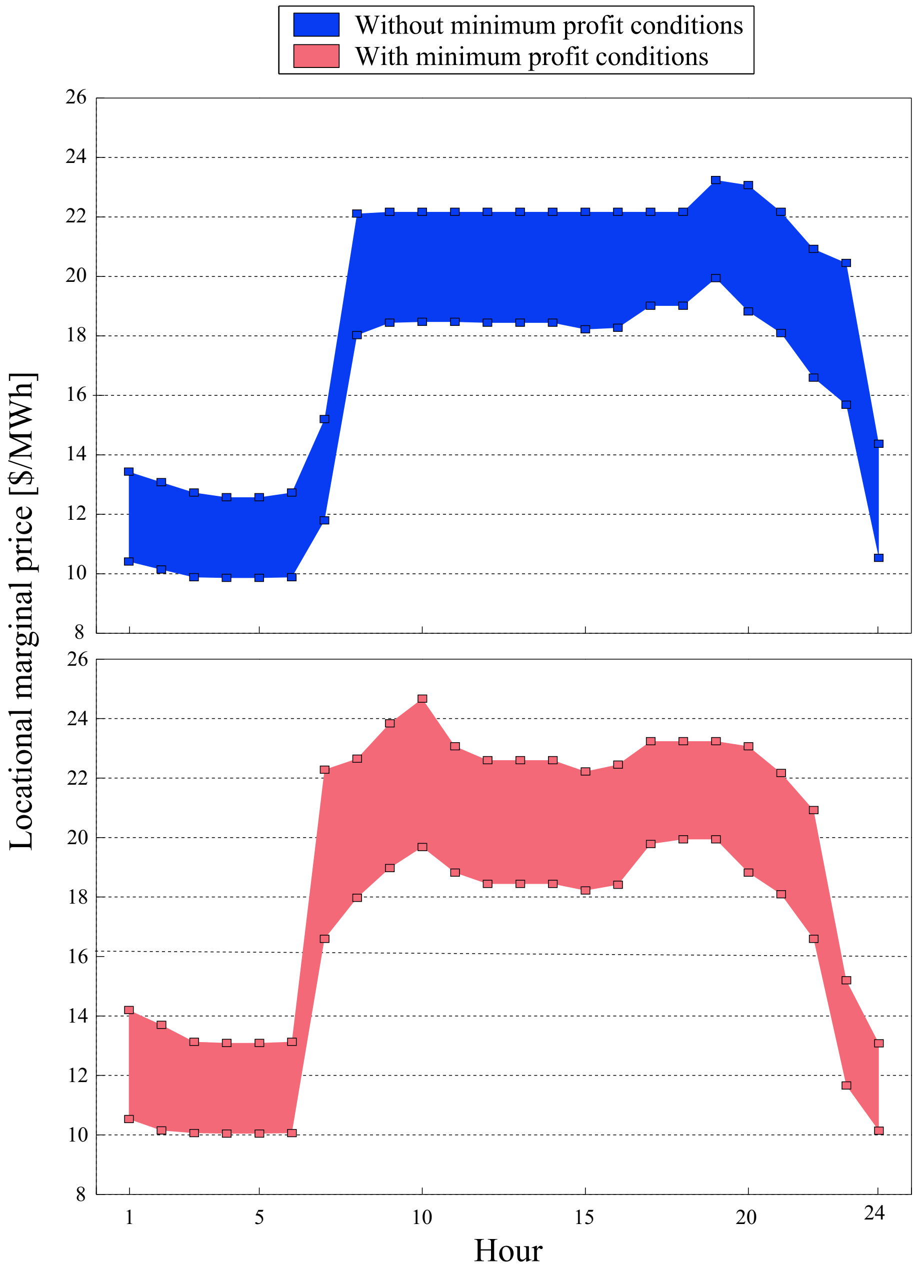

Figure 5.10: Comparison in terms of LMP. Multi-period equilibrium with and without MPC

Figure 5.11 compares generating profits for the multi-period equilibrium with and without minimum profit conditions. Note that almost all generating units experience increments in profits except units 9 and 30. These increments are due to the increments in locational marginal prices as can be seen in Figure 5.10. The profit of generating unit 9 is equal to zero because this unit has been expelled from the market for not satisfying minimum profit requirements.

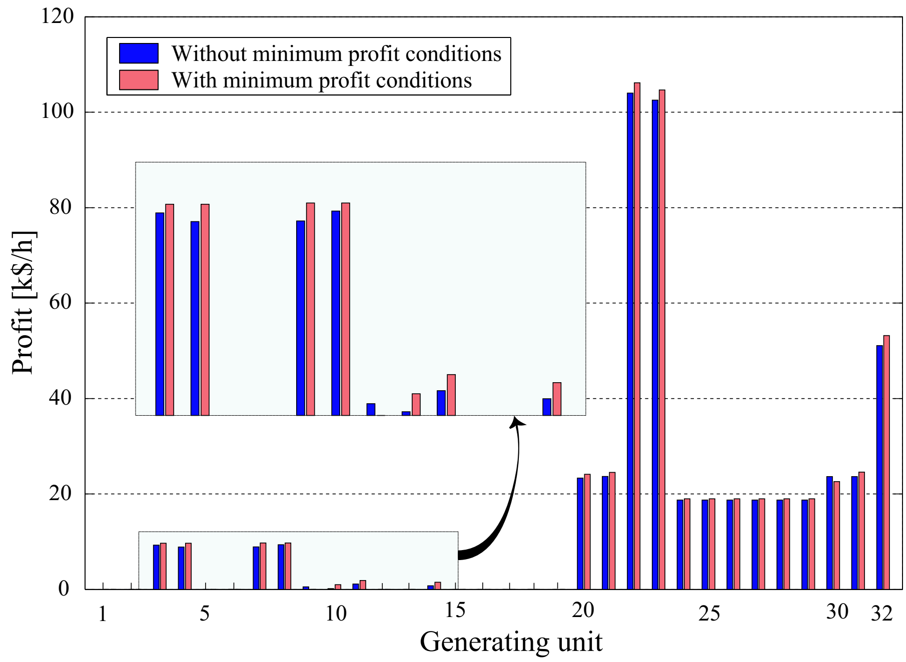

Figure 5.11: Comparison in terms of profit. Multi-period equilibrium with and without MPC

Figure 5.12 shows a comparison of the multi-period equilibrium not including and including minimum profit conditions in terms of demand costs. Note that demand costs increase for each demand except for demand 7. This is due to the generalized increments in locational marginal prices.

Table 5.19 provides a comparison of the multi-period equilibrium with and without minimum profit conditions. The last column shows variation of both cases with respect to the case without minimum profit conditions. We observe that the consideration of minimum profit conditions implies an increase in producer revenues and, therefore, an increase in the demand cost, whose consequence is higher producer surplus and lower consumer surplus. Social welfare, if minimum profit conditions are considered, is lower than if it is not.

Figure 5.13 illustrates the economic metrics for both multi-period equilibrium with and without minimum profit conditions. The main differences

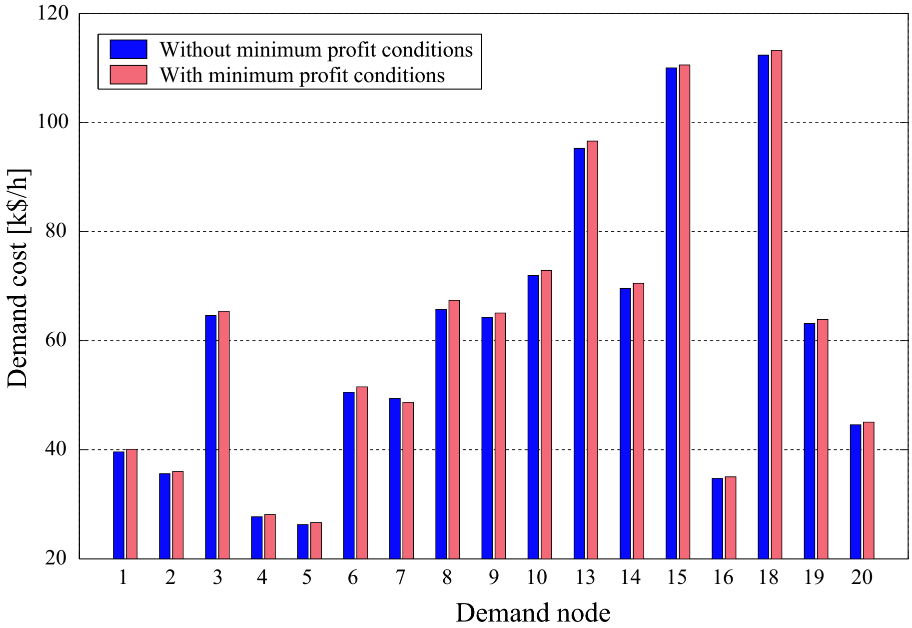

Figure 5.12: Comparison in terms of demand cost. Multi-period equilibrium with and without MPC

Table 5.19: Comparison of relevant metrics. Multi-period equilibrium with and without MPC

	Multi-period equilibrium without MPC	Multi-period equilibrium with MPC	Difference [%]
Total energy produced [MWh]	58242.44	58009.30	-0.40
Total energy consumed [MWh]	56566.85	56306.04	-0.46
Producer revenues [k$]	1003.81	1014.04	1.02
Demand costs [k$]	1025.51	1036.96	1.12
Star-up costs [k$]	3.06	2.79	-8.82
Producer surplus [k$]	503.20	516.88	2.72
Consumer surplus [k$]	237.44	220.74	-7.03
Merchandising surplus [k$]	21.70	22.92	5.62
Social welfare [k$]	762.34	760.54	-0.24
Declared social welfare [k$]	767.79	765.57	-0.29

occur in the producer and consumer surplus. The first one is higher for the case with minimum profit conditions and the second one is lower for the minimum profit constrained equilibrium.

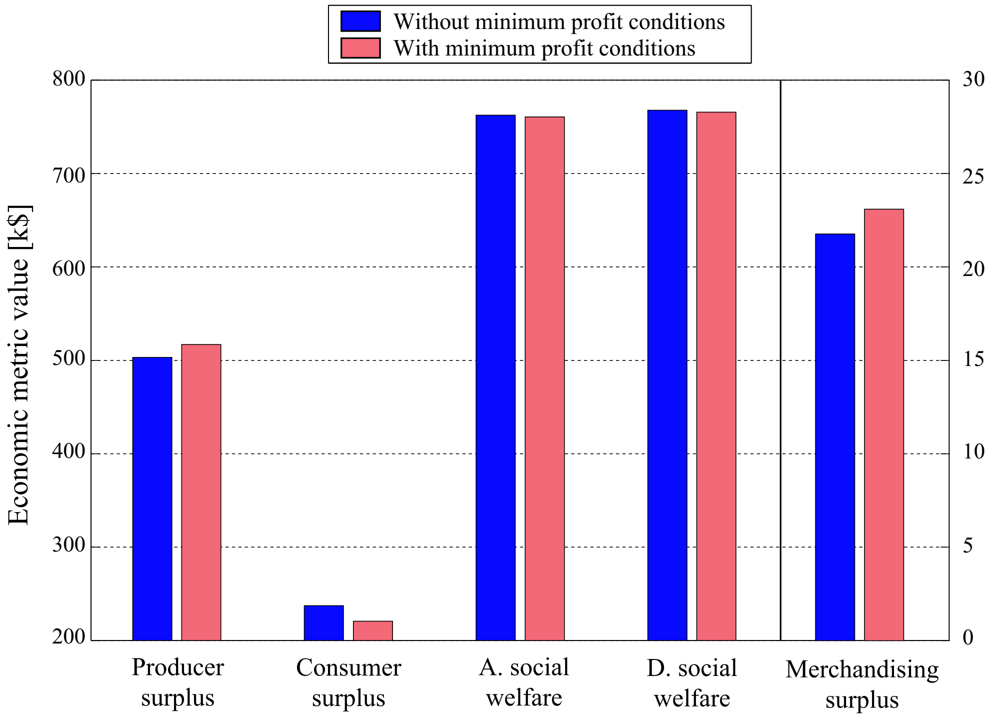

Figure 5.13: Comparison in terms of economic metrics. Multi-period equilibrium with and without MPC

5.3.5 Evolution of the Multi-Period Equilibrium as Minimum Profit Conditions Change

We obtain the multi-period equilibrium for several cases in which the generating units impose increasing minimum profit requirements. As regards case 1, generating units 3 and 4 each impose a minimum profit requirement of $ 9000, generating units 9, 10 and 11 of $ 200 and the rest of the units impose a minimum profit requirement of $ 0. This case corresponds to the one solved in Subsection 5.3.3. As concerns case 2, generating units 3, 4, 7 and 8 each impose a minimum profit of $ 9000, units 9, 10 and 11 of $ 1000, units 12, 13 and 14 of $ 1500 and the rest of the units require a nonnegative minimum profit. Finally, with regard to case 3, generating unit 7 imposes a minimum profit of $ 9000, generating units 3, 4 and 8 each impose minimum profit of $ 10000, units 9, 10 and 11 of $ 1200, units 12, 13 and 14 of $ 1500, units 20, 21, 30 and 31 of $ 25000 and the rest of the units impose a nonnegative minimum profit. Note that for each case, minimum profit conditions for the units are more restrictive than those imposed in the previous case. Table 5.20 provides the minimum profit conditions imposed by each generating unit for each case.

Figure 5.14 illustrates the variation range of locational marginal prices

Table 5.20: Minimum profit requirements imposed by each unit for each multi-period case

Generating unit	Case 1 [k$]	Case 2 [k$]	Case 3 [k$]
1, 2	0.0	0.0	0.0
3, 4	9.0	9.0	10.0
5, 6	0.0	0.0	0.0
7	0.0	9.0	9.0
8	0.0	9.0	10.0
9-11	0.2	1.0	1.2
12-14	0.0	1.5	1.5
15-19	0.0	0.0	0.0
20, 21	0.0	0.0	25.0
22-29	0.0	0.0	0.0
30, 31	0.0	0.0	25.0
32	0.0	0.0	0.0

for each hour for the three cases. The maximum and minimum locational marginal price for each hour of each case is represented by a small square. Prices in hours 7-10 are generally higher for case 1 than for case 2. On the other hand, in hours 23 and 24, locational marginal prices are higher for case 2. For the rest of the hours, prices for cases 1 and 2 are similar. As regards case 3, we can see that maximum price values are higher than for the other cases except for hours 7-10. Higher prices in case 3 are due to the more stringent minimum profit conditions in this case.

Figure 5.15 represents generating profits for the three cases. Case 2 is first compared with case 1. Generating unit 9 is started up and the profit of unit 11 decreases, and unit 12 is also started up but unit 14, which is located at the same node as unit 12, is shut down. We observe a profit decrease for unit 20 while the profit increases for the rest of the units. These changes for case 2 are caused by the more restrictive minimum profit conditions, which increase power produced in the system as well as some locational marginal prices to satisfy the minimum profit requirements. For case 3, the main differences with respect to case 2 are related to units 9 and 14. Unit 9 is shut down and unit 14 is started up. This might result from the fact that the minimum profit condition for unit 9 is high as it is more economical to shut down this unit and to compensate the subsequent power decrease by increasing the production of unit 14. For the rest of the units we observe a profit increment due to the generating power and price increments.

Figure 5.16 compares demand costs for the three cases which consider different minimum profit requirements. Demand costs increase as minimum

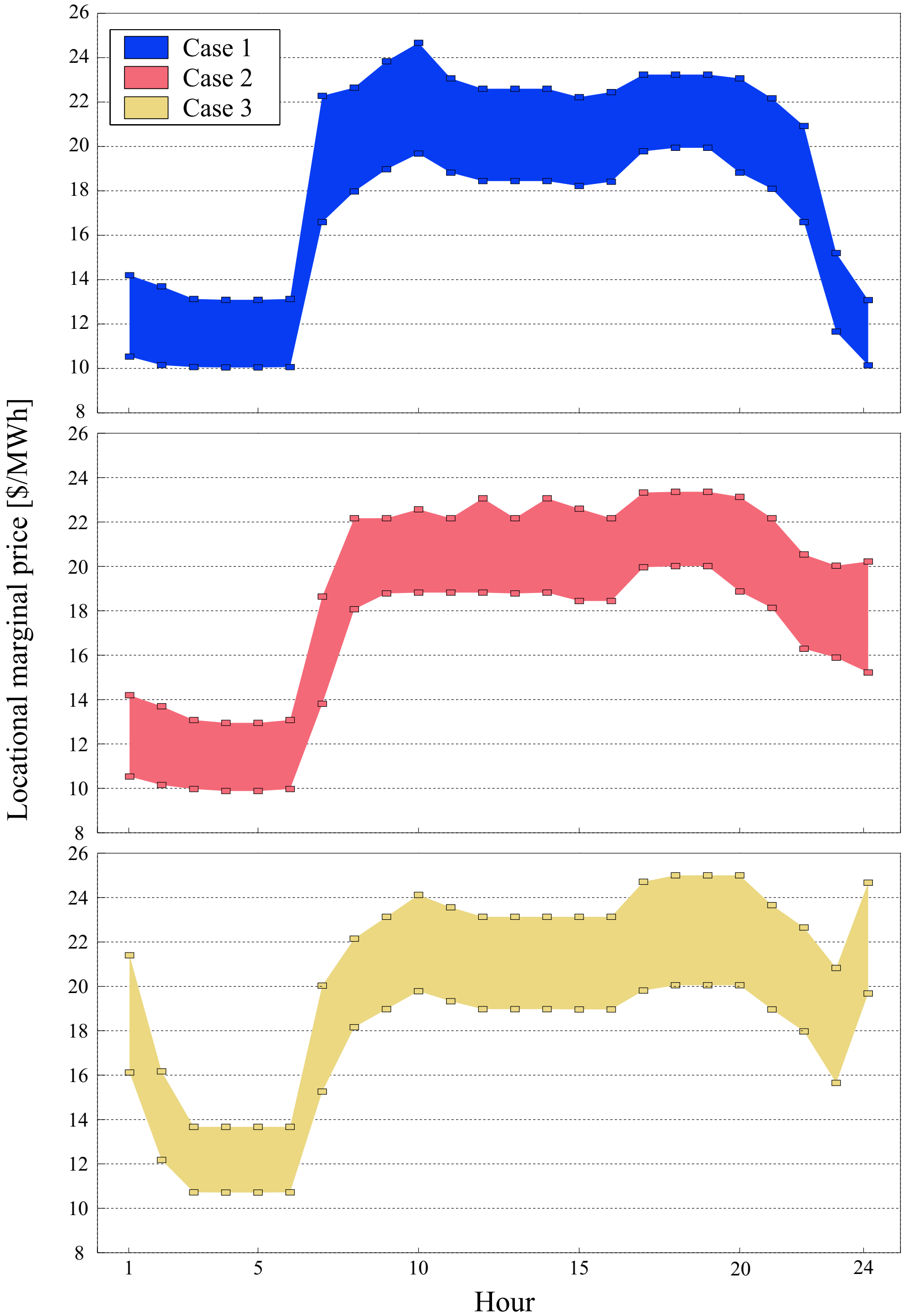

Figure 5.14: Comparison in terms of LMP. Evolution of the multi-period equilibrium

profit requirements are more restrictive for every demand. One reason for this evolution is that the power consumed in the system increases as minimum profit requirements are more restrictive.

Table 5.21 provides an economic comparison of the three cases. In the

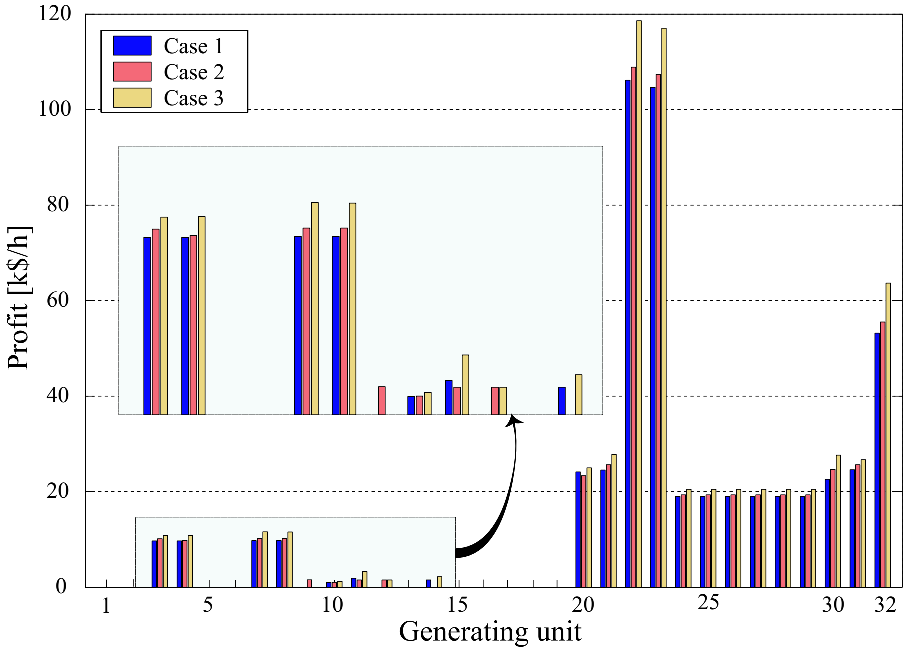

Figure 5.15: Comparison in terms of profit. Evolution of the multi-period equilibrium

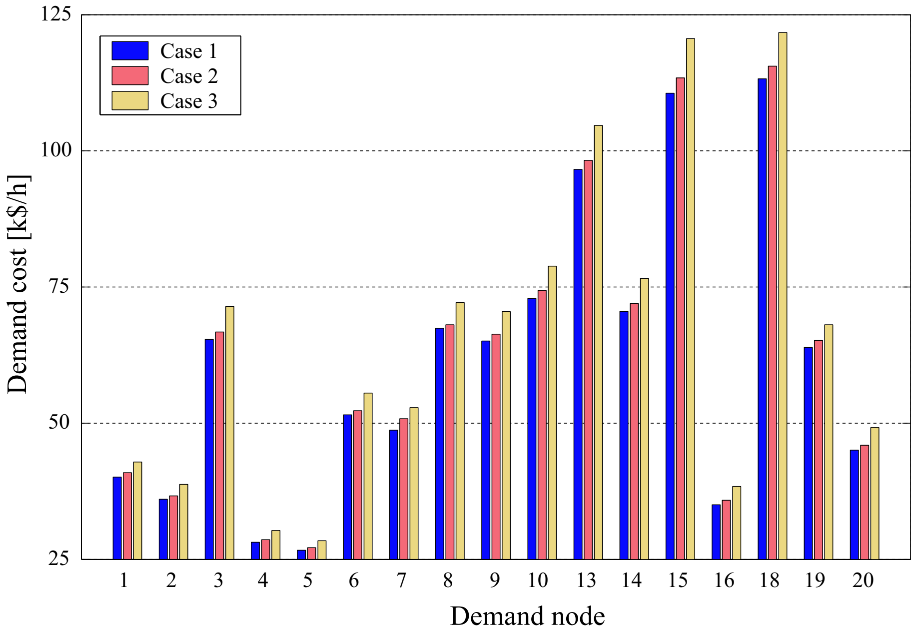

Figure 5.16: Comparison in terms of demand cost. Evolution of the multi-period equilibrium

last two columns, we find the percentage difference between case 2 and case 1 with respect to case 1, and between case 3 and case 2 with respect to case 2. As minimum profit conditions become more restrictive, the power produced and consumed in the system increases. The same takes place for producer revenues and demand costs. Start-up costs are higher as minimum profit conditions become more restrictive because the production schedule changes, resulting in lower operating costs and higher start-up costs for some generating units. Note that the producer surplus increases and the consumer surplus decreases as we impose higher minimum profit conditions. Differences in social welfare are low.

Table 5.21: Comparison of relevant metrics. Evolution of the multi-period equilibrium

	Case 1	Case 2	Case 3	2 versus 1 [%]	3 versus 2 [%]
Total energy produced [MWh]	58009.30	58334.71	58766.21	0.56	0.74
Total energy consumed [MWh]	56306.04	56656.52	57084.49	0.62	0.76
Producer revenues [k$]	1014.04	1034.69	1098.53	2.04	6.17
Demand costs [k$]	1036.96	1058.01	1120.21	2.03	5.88
Start-up costs [k$]	2.79	3.06	4.12	9.68	34.64
Producer surplus [k$]	516.88	532.61	581.51	3.04	9.18
Consumer surplus [k$]	220.74	206.57	153.25	-6.42	-25.81
Merchandising surplus [k$]	22.92	23.38	22.25	2.01	-4.83
Social welfare [k$]	760.54	762.56	757.01	0.27	-0.73
Declared social welfare [k$]	765.57	767.96	763.45	0.31	-0.59

Finally, Figure 5.17 illustrates the considered economic metrics for the three multi-period equilibrium cases. Note how producer and consumer surplus increases and decreases, respectively. These changes are more apparent in case 3.

5.3.6 Problem Size

Table 5.22 provides the size of the mixed-integer linear programming problem, the master problem, and the continuous quadratic programming problem, the subproblem, solved to obtain the multi-period equilibrium with and without minimum profit conditions.

The master problem is solved employing the solver GAMS / CPLEX 9.0 and the subproblem using GAMS / MINOS 5.51. The relative tolerance of the Benders decomposition algorithm is 0.01 for all cases. The computer used is a Dell PowerEdge 6600 with 4 processors at 1.60 GHz and 2 GB of RAM memory. Table 5.23 provides the CPU time required to solve the

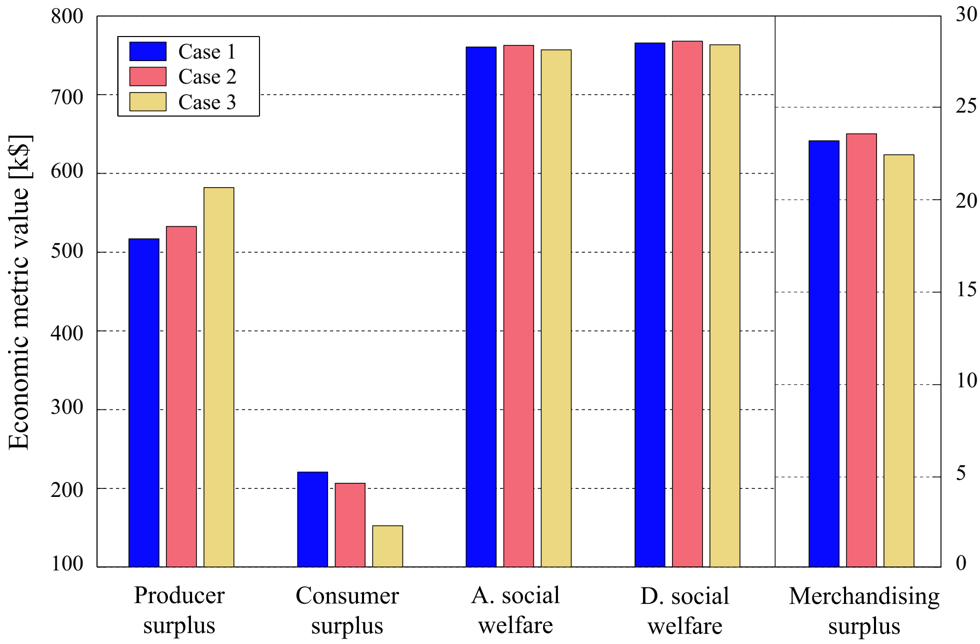

Figure 5.17: Comparison in terms of economic metrics. Evolution of the multi-period equilibrium

Table 5.22: Size of problems for multi-period equilibrium models

	Multi-period without MPC		Multi-period with MPC	
	Master problem	Subproblem	Master problem	Subproblem
Number of continuous variables	1537	50520	1537	50520
Number of binary variables	768	—	768	—
Number of constraints	3153	92736	3153	92768

multi-period equilibrium problems studied in this section and the infeasibility cost produced in each case. The infeasibility cost increases as minimum profit conditions are more restrictive. Infeasibility costs of case 2 and 3 are originated by the fact that some demands are paying a price higher than their utility. These demands are compensated by their respective economic losses, and the incurred cost required to compensate demands is distributed among all the market participants. The last column of the table represents the percentage of the infeasibility cost with respect to the respective demand total cost. Note finally that infeasibility costs are insignificant in percent.

Table 5.23: CPU time and infeasibility cost for multi-period equilibrium models

	CPU time [minutes]	Infeasibility cost [$]	Percentage [%]
No MPC	130	0	0.000
Case 1	128	0	0.000
Case 2	122	93	0.009
Case 3	295	1147	0.102

5.4 Summary

Several case studies based on the 24-node IEEE RTS are studied to illustrate the proposed models in Chapters 3 and 4. We compute the single-period equilibrium with and without minimum profit conditions for several market scenarios, and compare the results. The most relevant conclusions are that including minimum profit conditions in the model can cause increase in the equilibrium prices and / or cause some units to be expelled from the market. Afterward, we compare a multi-period equilibrium without considering minimum profit conditions and a succession of single-period equilibria concluding that the multi-period equilibrium approach reproduces a real-world functioning of the market in a better manner since coupling conditions are properly taken into account. Finally, we illustrate the effect of imposing minimum profit conditions on the multi-period equilibrium and conclude that minimum profit conditions can be satisfied by increasing the equilibrium prices, expelling some units from the market or reorganizing unit schedules, which minimizes the infeasibility of the problem.

Chapter 6

Conclusions and Future Research

This chapter presents an overview of the work reported in this document and the main conclusions of this dissertation. Then, the most important contributions of this work are provided and possible future research directions are suggested.

6.1 Thesis Summary

The work developed throughout this thesis can be summarized in the following items.

a) First, we propose an equilibrium procedure that coordinates the point of view of every market agent resulting in an equilibrium that simultaneously maximizes the independent objective of every market agent and satisfies network constraints. Therefore, the activities of the generating companies, consumers and an independent system operator are modeled:

   - The generating companies seek to maximize net profits by specifying hourly step functions of productions and minimum selling prices, and bounds on productions.
   - The goals of the consumers are to maximize their economic utilities by specifying hourly step functions of demands and maximum buying prices, and bounds on demands.
   - The independent system operator then clears the market taking into account consistency conditions as well as capacity and line losses so as to achieve maximum social welfare.

b) Then, we approach this equilibrium problem using complementarity theory in order to have the capability of imposing constraints on dual variables, i.e. on prices, such as minimum profit conditions for the generating units or maximum cost conditions for the consumers. In this way, given the form of the individual optimization problems, the Karush-Kuhn-Tucker conditions for the generating companies, the consumers and the independent system operator are both necessary and sufficient. The simultaneous solution to all these conditions constitutes a mixed linear complementarity problem.

c) Next, we include minimum profit constraints imposed by the generating units in the market equilibrium model. These constraints are added as additional constraints to the equivalent quadratic programming problem of the mixed linear complementarity problem of item b).

d) For the sake of clarity, the proposed equilibrium or near-equilibrium is first developed for the particular case considering only one time period. Afterwards, we consider an equilibrium or near-equilibrium applied to a multi-period framework. This model embodies binary decisions, i.e. on / off status for the units, and therefore optimality conditions cannot be directly applied. To avoid limitations provoked by binary variables, while retaining the advantages of using optimality conditions, we define the multi-period market equilibrium using Benders decomposition, which allows computing binary variables through the master problem and continuous variables through the subproblem.

e) Finally, we illustrate these market equilibrium concepts through several case studies.

6.2 Conclusions

The most relevant conclusions are enumerated below.

a) The multi-period equilibrium or near-equilibrium model developed in this thesis computes a solution in which every market agent maximizes its profit, all technical constraints are satisfied, and every scheduled generating unit meets its minimum profit requirement.

b) In the case that equilibrium prices do not exist, this near-equilibrium procedure generates prices that entail slight infeasibilities and that have a defendable interpretation as equilibrium prices.

c) A procedure to identify the equilibrium or near-equilibrium of an electricity market is of interest for market regulators that may use it for market monitoring. It is also of interest for the generating companies and the consumers to analyze their most appropriate strategies.

d) We show that the proposed single-period market equilibrium is equivalent to an optimal power flow in a centralized setting if minimum profit conditions for the generating units are not considered. However, unlike the optimal power flow, the proposed approach allows incorporating price-related constraints, such as minimum profit requirements, for online generating units in either a single-period or a multi-period framework. This fact is relevant in actual markets and represents an important modeling advantage.

e) The multi-period equilibrium approach reproduces a real-world functioning of the market in a better manner than a succession of single-period equilibria since coupling conditions are properly taken into account.

f) Including minimum profit conditions generally results in higher producer surplus and lower consumer surplus, while the social welfare does not change significantly.

g) This thesis also provides a methodology to solve quadratic programming problems with bilinear constraints (the minimum profit constraints) whose practical significance is that instead of needing specialized algorithms for bilinear problems, standard optimization solvers can be applied, thus facilitating computations with existing approaches.

h) Finally, we can conclude that Benders decomposition method allows a multi-period equilibrium which includes indivisibilities, i.e., nonconvexities, to be solved efficiently.

6.3 Contributions

The main contributions of this work can be summarized as follows:

a) The computation of equilibrium prices or near-equilibrium prices, if the former do not exist, in a pool-based electricity market.

b) The formulation of a multi-period market equilibrium procedure that includes constraints on prices (dual variables), and more specifically, minimum profit conditions for the generating units. These minimum profit requirements may render a generating unit uncompetitive and expel it from the market.

c) The equilibrium model includes both line capacity limits and linearized losses. Therefore, we can use locational marginal prices, and take into account the effects of line congestion and transmission losses in an accurate and efficient manner.

d) The formulation of two methodologies to solve the linear and quadratic programming problems that involve bilinear constraints. The first methodology approximates the bilinear equations using Schur's decomposition and binary variables. The second one obtains the solution through an iterative procedure based on the successive over-relaxation iterative method.

e) Combination of complementarity theory and Benders decomposition to achieve the solution of complex mixed integer complementarity problems.

f) The formulation of a methodology based on Benders decomposition to solve multi-period problems with indivisibilities (non-convexities), optimizing not only the continuous decisions, but also the status (binary) decisions.

g) The publication of the following six papers in journals of high international reputation. Note that the first, fourth, fifth and sixth papers are directly related to this thesis, while the rest are the result of collateral researches.

- A. J. Conejo, F. D. Galiana, J. M. Arroyo, R. García-Bertrand, Cheong Wei Chua and M. Huneault, *"Economic Inefficiencies and Cross-Subsidies in an Auction-Based Electricity Pool"*. IEEE Transactions on Power Systems, vol. 18, no. 1, pages 221-228, February 2003.
- A. J. Conejo, F. J. Nogales, J. M. Arroyo and R. García-Bertrand, *"Risk-Constrained Self-Scheduling of a Thermal Power Producer"*. IEEE Transactions on Power Systems, vol. 19, no. 3, pages 1569-1574, August 2004.
- A. J. Conejo, R. García-Bertrand and M. Díaz-Salazar, *"Generation Maintenance Scheduling in Restructured Power Systems"*. IEEE Transactions on Power Systems, vol. 20, no. 2, pages 984-992, May 2005.
- R. García-Bertrand, A. J. Conejo and S. Gabriel, *"Electricity Market Near-Equilibrium under Locational Marginal Pricing and Minimum Profit Conditions"*. European Journal of Operational Research, in press 2005.
- S. A. Gabriel, R. García-Bertrand, P. Sahakij and A. J. Conejo, *"A Practical Approach to Approximate Bilinear Functions in Mathematical Programming Problems by Using Schur's Decomposition and SOS Type 2 Variables"*. The Journal of the Operational Research Society, in press 2005.

- R. García-Bertrand, A. J. Conejo and S. Gabriel, *"Multi-Period Near-Equilibrium in a Pool-Based Electricity Market Including On / Off Decisions"*. Networks and Spatial Economics, in press 2005.

h) Publication as a coauthor of a book in Springer. This book includes the Benders decomposition technique used throughout this thesis.

- A. J. Conejo, E. Castillo, R. Mínguez and R. García-Bertrand, *"Decomposition Techniques in Mathematical Programming. Engineering and Science Applications"*. Springer, in press.

6.4 Future Work

Suggestions for future work resulting from the work reported in this thesis are listed below:

a) To formulate the problem of a generating company in further detail, modeling non-convex and non-differentiable operating costs, exponential start-up costs, available spinning reserve taking into account ramp rate restrictions, and minimum up and down time constraints; and also extending the model to include hydroelectric generating units.

b) To further characterize and model demands.

c) To incorporate security constraints into the independent system operator problem, including active and reactive power margins and voltage limits, and using both deterministic and stochastic criteria.

d) To use nonlinear complementarity problems for modeling the behavior of the market agents.

e) In this work, the network is modeled through a DC power flow including linearized losses. A more realistic representation of the network could be implemented using an AC power flow that implicity includes losses and reactive power.

f) The multi-period equilibrium model is solved using Benders decomposition algorithm. In this algorithm, the most important computational burden is related to the solutions of the master problem. Therefore, a relevant extension would be to study specialized mechanisms, heuristic or not, to efficiently solve this master problem.

g) To further analyze and characterize problems where formulations require the use of Benders decompositions.

Appendix A

Linear Complementarity Problem

A.1 Introduction

In an electricity market competitive equilibrium, market agents simultaneously optimize the production and consumption of electric energy, while the production and consumption are balanced. We use optimality conditions, that is, the complementarity theory, to simultaneously solve the problem faced by every market agent. This set of optimality conditions corresponds to a complementarity problem.

This appendix defines a linear complementarity problem and a mixed linear complementarity problem. Then, we state the equivalence between an equilibrium problem and a mixed linear complementarity problem. Finally, the way to formulate a mixed linear complementarity problem as a global optimization problem is shown.

A.2 Definition of Linear Complementarity Problem

The Linear Complementarity Problem (LCP) [25, 71] consists of finding vector $z \in \mathbb{R}^n$ such that,

$$
\begin{aligned}
z &\geq 0 && \text{(A.1)}\\
q + Mz &\geq 0 && \text{(A.2)}\\
z^{\mathrm{T}}(q + Mz) &= 0, && \text{(A.3)}
\end{aligned}
$$

for a given vector $q \in \mathbb{R}^n$ and a matrix $M \in \mathbb{R}^{n \times n}$.

Equation (A.3) is called the complementarity condition of the LCP.

The LCP can be rewritten more compactly as

$$
0 \leq q + Mz \perp z \geq 0, \tag{A.4}
$$

where symbol $\perp$ indicates that both inequalities are complementarity, that is, they satisfy the complementarity condition (A.3).

The LCP admits a number of interesting generalizations [25, 32, 71]. One of these is the Mixed Linear Complementarity Problem (MLCP) defined below. Let M_{11} and M_{22} be real square matrices of order n and m, respectively. Let $M_{12} \in \mathbb{R}^{n\times m}$, $M_{21} \in \mathbb{R}^{m\times n}$, $q_1 \in \mathbb{R}^n$ and $q_2 \in \mathbb{R}^m$ be given. The MLCP is to find vectors $z_1 \in \mathbb{R}^n$ and $z_2 \in \mathbb{R}^m$ such that,

$$
\begin{aligned}
q_1 + M_{11}z_1 + M_{12}z_2 &\geq 0 & \text{(A.5)}\\
q_2 + M_{21}z_1 + M_{22}z_2 &= 0 & \text{(A.6)}\\
z_1 &\geq 0 & \text{(A.7)}\\
z_1^{\mathrm{T}}\big(q_1 + M_{11}z_1 + M_{12}z_2\big) &= 0. & \text{(A.8)}
\end{aligned}
$$

Note that variable z_2 is not restricted to be nonnegative. Thus, the MLCP is a mixture of the LCP and a system of equations.

If matrix M_{11} is nonsingular, this MLCP can be converted into a standard LCP, therefore the MLCP can be treated as a standard LCP.

A.3 Equilibrium Problem as an MLCP

A market equilibrium is achieved when each generating company maximizes its profit, each consumer maximizes its utility and the independent system operator maximizes the social welfare.

The problem faced by each generating company can be compactly formulated as follows.

Maximize

$$p^{\mathrm{T}}x - c^{\mathrm{T}}x \tag{A.9}$$

subject to

$$
\begin{aligned}
b - Ax &\leq 0: \ \alpha & \text{(A.10)}\\
-x &\leq 0: \ \beta, & \text{(A.11)}
\end{aligned}
$$

where x represents the production levels, p the energy prices and c the production costs. Equation (A.9) is the profit of the generating company for selling energy x. Equation (A.10) represents the operating constraints for production devices whose dual variable is α. Equation (A.11) states that production levels are positive.

The following problem represents the behavior of each consumer.

Maximize

$$u^{\mathrm{T}}y - p^{\mathrm{T}}y \tag{A.12}$$

subject to

$$
\begin{aligned}
e - Dy &\leq 0: \ \mu & \text{(A.13)}\\
-y &\leq 0: \ \gamma, & \text{(A.14)}
\end{aligned}
$$

where y represents the consumption levels and u the marginal utility. Equation (A.12) is the utility of the consumer for buying energy y. Equation (A.13) represents the demand requirements where μ is the corresponding dual variable. Equation (A.14) states that consumption levels are positive.

Finally, the independent system operator balances the market maximizing the social welfare as expressed by the following problem.

Maximize

$$f^{\mathrm{T}}y - g^{\mathrm{T}}x \tag{A.15}$$

subject to

$$h - Ix - Jy \;=\; 0 : \;\rho, \tag{A.16}$$

where dual variable ρ represents the energy price. Equation (A.15) is the social welfare. Equation (A.16) represents the balance constraints.

The solution of the equilibrium problem can be obtained by solving the first order optimality conditions of the problems faced by each generating company, each consumer and the ISO. This set of optimality conditions is set out below.

$$\begin{aligned}
-p + c - A^{\mathrm{T}}\alpha - \beta &= 0 && \text{(A.17)}\\
(b - Ax)\alpha &= 0 && \text{(A.18)}\\
(-x)\beta &= 0 && \text{(A.19)}\\
b - Ax &\leq 0 && \text{(A.20)}\\
-x &\leq 0 && \text{(A.21)}\\
\alpha &\geq 0 && \text{(A.22)}\\
\beta &\geq 0 && \text{(A.23)}\\
-u + p - D^{\mathrm{T}}\mu - \gamma &= 0 && \text{(A.24)}\\
(e - Dy)\mu &= 0 && \text{(A.25)}\\
(-y)\gamma &= 0 && \text{(A.26)}\\
e - Dy &\leq 0 && \text{(A.27)}\\
-y &\leq 0 && \text{(A.28)}\\
\mu &\geq 0 && \text{(A.29)}\\
\gamma &\geq 0 && \text{(A.30)}\\
g - I^{\mathrm{T}}\rho &= 0 && \text{(A.31)}\\
-f - J^{\mathrm{T}}\rho &= 0 && \text{(A.32)}\\
h - Ix - Jy &= 0, && \text{(A.33)}
\end{aligned}$$

where equations (A.17)-(A.23) are the optimality conditions of the problem of the generating companies, equations (A.24)-(A.30) are the optimality conditions of the problem of the consumers, and equations (A.31)-(A.33) are the optimality conditions of the problem of the independent system operator. The above system of equations can be reduced by eliminating variables β and γ, and replacing variable ρ by p. The resulting system is,

$$\begin{aligned}
(-p + c - A^{\mathrm{T}}\alpha)x &= 0 && \text{(A.34)}\\
-p + c - A^{\mathrm{T}}\alpha &\geq 0 && \text{(A.35)}\\
(Ax - b)\alpha &= 0 && \text{(A.36)}\\
Ax - b &\geq 0 && \text{(A.37)}\\
x &\geq 0 && \text{(A.38)}\\
\alpha &\geq 0 && \text{(A.39)}\\
(-u + p - D^{\mathrm{T}}\mu)y &= 0 && \text{(A.40)}\\
-u + p - D^{\mathrm{T}}\mu &\geq 0 && \text{(A.41)}\\
(Dy - e)\mu &= 0 && \text{(A.42)}\\
Dy - e &\geq 0 && \text{(A.43)}\\
y &\geq 0 && \text{(A.44)}\\
\mu &\geq 0 && \text{(A.45)}\\
g - I^{\mathrm{T}}p &= 0 && \text{(A.46)}\\
-f - J^{\mathrm{T}}p &= 0 && \text{(A.47)}\\
h - Ix - Jy &= 0. && \text{(A.48)}
\end{aligned}$$

The above system of inequalities and equalities constitutes an MLCP where

$$z_1 = \begin{pmatrix} x \\ \alpha \\ y \\ \mu \end{pmatrix}, \qquad z_2 = p,$$

$$q_1 = \begin{pmatrix} c \\ -b \\ -u \\ -e \end{pmatrix}, \qquad q_2 = \begin{pmatrix} g \\ -f \\ h \end{pmatrix},$$

$$M_{11} = \begin{pmatrix} 0 & -A^{\mathrm{T}} & 0 & 0 \\ A & 0 & 0 & 0 \\ 0 & 0 & 0 & -D^{\mathrm{T}} \\ 0 & 0 & D & 0 \end{pmatrix}, \qquad M_{12} = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$M_{21} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -I & 0 & -J & 0 \end{pmatrix} \quad \text{and} \quad M_{22} = \begin{pmatrix} -I^{\mathrm{T}} \\ -J^{\mathrm{T}} \\ 0 \end{pmatrix}.$$

A.4 Formulation of an MLCP as a Global Optimization Problem

A large number of methods for solving LCP has been proposed [25, 71]. Many of these methods are based on the assumption that matrix M belongs to a particular class of matrices, distinguishing the positive semi-definite and P-matrices (all its principal minors are positive) as the most interesting of these classes. If M is positive semi-definitive and linear constraints are consistent, the LCP always has a solution. If M is a P-matrix, the LCP has a unique solution. If the matrix M belongs to one of these classes of matrices, the LCP can also be found as a stationary point of the following quadratic programming problem [25, 59, 71].

Minimize

$$z^{\mathrm{T}}(q + Mz) \tag{A.49}$$

subject to

$$\begin{aligned} z &\geq 0 & \text{(A.50)}\\ q + Mz &\geq 0. & \text{(A.51)} \end{aligned}$$

In general, a solution of the LCP requires the computation of a global minimum of this quadratic programming problem. However, vector z is a solution of the LCP if and only if it is a global minimum of this problem with an objective function value of zero. Note that the objective function (A.49) is always bounded below by zero on the feasible set.

For an MLCP, the equivalent quadratic programming problem is, minimize

$$z_1^{\mathrm{T}}\left(q_1 + M_{11}z_1 + M_{12}z_2\right) \tag{A.52}$$

subject to

$$\begin{aligned} z_1 &\geq 0 & \text{(A.53)}\\ q_1 + M_{11}z_1 + M_{12}z_2 &\geq 0 & \text{(A.54)}\\ q_2 + M_{21}z_1 + M_{22}z_2 &= 0, & \text{(A.55)} \end{aligned}$$

where the optimal objective function value must be equal to zero.

Appendix B

Benders Decomposition

Benders decomposition [21] algorithm allows a nonlinear programming problem to be solved with complicating variables in a distributed manner at the cost of iteration. The complicating variables are those variables that make the solution of the problem difficult to solve. If they are fixed to given values, the problem becomes substantially simpler. Benders decomposition is described in this appendix.

B.1 Description

The mixed-integer nonlinear programming problem considered is

$$\underset{x_1,\ldots,x_n;y_1,\ldots,y_m}{\text{minimize}} \quad f_1(x_1,\ldots,x_n;y_1,\ldots,y_m) + f_2(x_1,\ldots,x_n) \tag{B.1}$$

subject to

$$h_k(x_1,\ldots,x_n;y_1,\ldots,y_m) = 0;\ k = 1,\ldots,q \tag{B.2}$$

$$g_l(x_1,\ldots,x_n;y_1,\ldots,y_m) \le 0;\ l = 1,\ldots,r \tag{B.3}$$

$$x_i^{\text{down}} \le x_i \le x_i^{\text{up}},\ x_i \in \mathbb{N};\ i = 1,\ldots,n \tag{B.4}$$

$$y_j^{\text{down}} \le y_j \le y_j^{\text{up}},\ y_j \in \mathbb{R};\ j = 1,\ldots,m. \tag{B.5}$$

Note that upper and lower bounds are imposed on optimization variables to reflect physical limits, which result in a simpler mathematical treatment.

It is assumed that the continuous nonlinear programming problem resulting from fixing in the original mixed-integer nonlinear programming problem the integer variables to given feasible values is convex; otherwise, the convergence of the procedures analyzed in this section cannot be guaranteed. However, local convexity in a neighborhood of the optimal solution is sufficient to guarantee convergence in most practical applications.

The solution of the problem (B.1)-(B.5) can be obtained by parameterizing this problem as a function of the complicating variables $x_1,\ldots,x_n$. This is done as follows.

We define the auxiliary function $\alpha(\boldsymbol{x})$ that expresses the objective function of the original problem as a function solely of the complicating variables,

$$\alpha(\boldsymbol{x}) = \underset{y_1,\dots,y_m}{\text{minimum}} \quad f_1(x_1,\dots,x_n;y_1,\dots,y_m) \tag{B.6}$$

subject to

$$h_k(x_1,\dots,x_n;y_1,\dots,y_m) = 0;\ k=1,\dots,q \tag{B.7}$$
$$g_l(x_1,\dots,x_n;y_1,\dots,y_m) \le 0;\ l=1,\dots,r \tag{B.8}$$
$$y_j^{\text{down}} \le y_j \le y_j^{\text{up}},\ y_j \in \mathbb{R};\ j=1,\dots,m. \tag{B.9}$$

Using function $\alpha(\boldsymbol{x})$, the original problem can be expressed as,

$$\underset{x_1,\dots,x_n}{\text{minimize}} \quad \alpha(\boldsymbol{x}) + f_2(x_1,\dots,x_n) \tag{B.10}$$

subject to

$$x_i^{\text{down}} \le x_i \le x_i^{\text{up}},\ x_i \in \mathbb{N};\ i=1,\dots,n. \tag{B.11}$$

The procedure explained below produces iteratively better and better approximations to function $\alpha(\boldsymbol{x})$. If complicating variables are fixed to specific values using constraints of the form, $x_i = x_i^{(\nu)}$, the resulting problem is easy to solve. This problem has the form

$$\underset{y_1,\dots,y_m}{\text{minimum}} \quad f_1(x_1,\dots,x_n;y_1,\dots,y_m) \tag{B.12}$$

subject to

$$h_k(x_1,\dots,x_n;y_1,\dots,y_m) = 0;\ k=1,\dots,q \tag{B.13}$$
$$g_l(x_1,\dots,x_n;y_1,\dots,y_m) \le 0;\ l=1,\dots,r \tag{B.14}$$
$$y_j^{\text{down}} \le y_j \le y_j^{\text{up}},\ y_j \in \mathbb{R};\ j=1,\dots,m \tag{B.15}$$
$$x_i = x_i^{(\nu)} :\ \lambda_i^{(\nu)};\ i=1,\dots,n. \tag{B.16}$$

The problem above is denominated a subproblem. Typically, it decomposes in many subproblems. The solution of the problem above provides values for the non-complicating variables, $y_j^{(\nu)}$ $(j=1,\dots,m)$, and the dual variable vector associated to those constraints that fix the complicating variables to given values. This sensitivity vector is denoted by $\lambda_i^{(\nu)}$ $(i=1,\dots,n)$. The information obtained solving the subproblem allows the original problem to be reproduced more and more accurately. Moreover, if function $\alpha(\boldsymbol{x})$ is convex, the following problem approximates the original one from below.

$$\underset{\alpha;x_1,\dots,x_n}{\text{Minimize}} \quad \alpha + f_2(x_1,\dots,x_n) \tag{B.17}$$

subject to

$$x_i^{\text{down}} \le x_i \le x_i^{\text{up}},\ x_i \in \mathbb{N};\ i = 1, \ldots, n \tag{B.18}$$

$$\alpha \ge f_1(x_1^{(\ell)}, \ldots, x_n^{(\ell)}; y_1^{(\ell)}, \ldots, y_m^{(\ell)}) + \sum_{i=1}^{n} \lambda_i^{(\ell)}(x_i - x_i^{(\ell)});$$
$$\ell = 1, \ldots, \nu - 1. \tag{B.19}$$

The last constraint of the problem above is called Benders cut. The problem itself is denominated a master problem. The solution of this master problem provides new values for the complicating variables that are used for solving a new subproblem. In turn, this subproblem provides information to formulate a more accurate master problem that provides new values of complicating variables. The procedure continues until upper and lower bounds of the objective function optimal value are close enough.

B.2 Bounds

It should be noted that problem (B.17)-(B.19) is a relaxed version of the original problem and its objective function approximates from below the objective function of the original problem. Therefore, for iteration ν, the optimal value of the objective function of problem (B.17)-(B.19) is a lower bound of the optimal value of the objective function of the original problem. Therefore,

$$z_{\text{down}}^{(\nu)} = \alpha^{(\nu)} + f_2(x_1^{(\nu)}, \ldots, x_n^{(\nu)}). \tag{B.20}$$

On the other hand, problem (B.12)-(B.16), the subproblem, is a further restricted version of the original problem. Therefore, its optimal objective function value is an upper bound of the optimal value of the objective function of the original problem, i.e.,

$$z_{\text{up}}^{(\nu)} = f_1(x_1^{(\nu)}, \ldots, x_n^{(\nu)}; y_1^{(\nu)}, \ldots, y_m^{(\nu)}) + f_2(x_1^{(\nu)}, \ldots, x_n^{(\nu)}). \tag{B.21}$$

B.3 The Benders Decomposition Algorithm

The Benders decomposition algorithm to solve mixed-integer nonlinear programming problems is described below.

Algorithm B.1 (The Benders decomposition algorithm to solve mixed-integer nonlinear programming problems).
Step 0: Initialization. Initialize the iteration counter, $\nu = 1$.

Solve the initial mixed-integer linear programming master problem below (this does not include Benders cuts).

$$\underset{\alpha;x_1,\dots,x_n}{\text{Minimize}} \quad \alpha + f_2(x_1,\dots,x_n) \tag{B.22}$$

subject to

$$x_i^{\text{down}} \le x_i \le x_i^{\text{up}},\ x_i \in \mathbb{N};\ i=1,\dots,n \tag{B.23}$$

$$\alpha \ge \alpha^{\text{down}}. \tag{B.24}$$

Its trivial solution is $x_1^{(\nu)},\dots,x_n^{(\nu)};\ \alpha^{(\nu)} = \alpha^{\text{down}}$.

Step 1: Subproblem solution. Solve the continuous nonlinear programming subproblem,

$$\underset{y_1,\dots,y_m}{\text{minimum}} \quad f_1(x_1,\dots,x_n;y_1,\dots,y_m) \tag{B.25}$$

subject to

$$h_k(x_1,\dots,x_n;y_1,\dots,y_m) = 0;\ k=1,\dots,q \tag{B.26}$$

$$g_l(x_1,\dots,x_n;y_1,\dots,y_m) \le 0;\ l=1,\dots,r \tag{B.27}$$

$$y_j^{\text{down}} \le y_j \le y_j^{\text{up}},\ y_j \in \mathbb{R};\ j=1,\dots,m \tag{B.28}$$

$$x_i = x_i^{(\nu)}:\ \lambda_i^{(\nu)};\ i=1,\dots,n. \tag{B.29}$$

The solution to this problem is $y_1^{(\nu)},\dots,y_m^{(\nu)}$, with dual variable values $\lambda_1^{(\nu)},\dots,\lambda_n^{(\nu)}$.

The problem above may decompose into independent subproblems that can be solved independently. This is a situation often encountered in practice.

Step 2: Convergence checking. Compute upper and lower bounds of the optimal value of the objective function of the original problem:

$$z_{\text{up}}^{(\nu)} = f_1(x_1^{(\nu)},\dots,x_n^{(\nu)};y_1^{(\nu)},\dots,y_m^{(\nu)}) + f_2(x_1^{(\nu)},\dots,x_n^{(\nu)}) \tag{B.30}$$

$$z_{\text{down}}^{(\nu)} = \alpha^{(\nu)} + f_2(x_1^{(\nu)},\dots,x_n^{(\nu)}). \tag{B.31}$$

If $z_{\text{up}}^{(\nu)} - z_{\text{down}}^{(\nu)}$ is smaller than a pre-specified tolerance, ε, stop, the optimal solution is $x_1^{(\nu)},\dots,x_n^{(\nu)}$ and $y_1^{(\nu)},\dots,y_m^{(\nu)}$. If this is not the case, the algorithm continues to the next step.

Step 3: Master problem solution. Update the iteration counter, $\nu \leftarrow \nu + 1$.

Solve the mixed-integer linear programming master problem,

$$\underset{\alpha;x_1,\dots,x_n}{\text{minimize}} \quad \alpha + f_2(x_1,\dots,x_n) \tag{B.32}$$

subject to

$$x_i^{\text{down}} \leq x_i \leq x_i^{\text{up}},\ x_i \in \mathbb{N};\ i = 1, \ldots, n \tag{B.33}$$

$$\alpha \geq f_1(x_1^{(\ell)}, \ldots, x_n^{(\ell)}; y_1^{(\ell)}, \ldots, y_m^{(\ell)}) + \sum_{i=1}^{n} \lambda_i^{(\ell)}(x_i - x_i^{(\ell)});$$

$$\ell = 1, \ldots, \nu - 1 \tag{B.34}$$

$$\alpha \geq \alpha^{\text{down}}. \tag{B.35}$$

The solution to this problem is $x_1^{(\nu)}, \ldots, x_n^{(\nu)}$ and $\alpha^{(\nu)}$. The algorithm continues to Step 1.

∎

Appendix C

Data: IEEE 24-Node Reliability Test System

C.1 Introduction

This appendix contains the data and description of the IEEE 24-node Reliability Test System (RTS) used for the simulations in Chapter 5. The details of this system can be found in [47].

C.2 Data for Generating Companies

Table C.1 provides the location of the generating units throughout the network as well as their respective capacities.

Table C.1: Generating unit locations and capacities

Node	Generating unit number (Unit size [MW])					
1	1 (20)	2 (20)	3 (76)	4 (76)		
2	5 (20)	6 (20)	7 (76)	8 (76)		
7	9 (100)	10 (100)	11 (100)			
13	12 (197)	13 (197)	14 (197)			
15	15 (12)	16 (12)	17 (12)	18 (12)	19 (12)	20 (155)
16	21 (155)					
18	22 (400)					
21	23 (400)					
22	24 (50)	25 (50)	26 (50)	27 (50)	28 (50)	29 (50)
23	30 (155)	31 (155)	32 (350)			

The number and size of the incremental heat rate blocks of the generating units are specified in Table C.2. Fuel costs have been taken from [9] and are 2.3 $/MBtu for #6 oil, 3.0 $/MBtu for #2 oil, 1.20 $/MBtu for coal, and 0.6

$/MBtu for nuclear. Using this information, marginal cost for each block of each unit can be obtained. They are provided in Table C.2.

Table C.2: Generating unit operating cost data

Unit size [MW]	Type	Fuel	Block size [MW]	Incremental heat rate [Btu/kWh]	Marginal cost [$/MWh]
12	Fossil steam	#6 oil	2.40	10179	23.41
			3.60	10330	23.78
			3.60	11668	26.84
			2.40	13219	30.40
20	Combustion turbine	#2 oil	15.80	9859	29.58
			0.20	10139	30.42
			3.80	14272	42.82
			0.20	14427	43.28
50	Hydro		50.00		0.00
76	Fossil steam	Coal	15.20	9548	11.46
			22.80	9966	11.96
			22.80	11576	13.89
			15.20	13311	15.97
100	Fossil steam	#6 oil	25.00	8089	18.60
			25.00	8708	20.03
			30.00	9420	21.67
			20.00	9877	22.72
155	Fossil steam	Coal	54.25	8265	9.92
			38.75	8541	10.25
			31.00	8900	10.68
			31.00	9381	11.26
197	Fossil steam	#6 oil	68.95	8348	19.20
			49.25	8833	20.32
			39.40	9225	21.22
			39.40	9620	22.13
350	Fossil steam	Coal	140.00	8402	10.08
			87.50	8896	10.66
			52.50	9244	11.09
			70.00	9768	11.72
400	Nuclear steam	LWR	100.00	8848	5.31
			100.00	8965	5.38
			120.00	9210	5.53
			80.00	9438	5.66

Table C.3 provides capacity, ramp rates, start-up and fixed costs for each generating unit. Shut-down costs are considered to be zero.

The size and bidding price of each block of each generating unit that

Table C.3: Unit capacity, ramp rates, and start-up and fixed costs

Unit size [MW]	Ramp rate [MW/h]	Start-up cost [$]	Fixed cost [$/h]
12	60	87.4	5.25
20	180	15.0	5.00
50	-	0.0	0.00
76	120	715.2	7.50
100	420	575.0	8.50
155	180	312.0	6.25
197	180	1018.9	15.00
350	240	2298.0	20.00
400	1200	0.0	0.00

each generating company is willing to sell are chosen to be the corresponding marginal cost values and are shown in Table C.4. These bids are assumed to be identical for all hours. The first block is considered to be the minimum power output. The sum of all blocks is the maximum power output. The hydro units are assumed to be run-of-the-river, thereby producing at maximum power the whole time horizon under study.

Table C.4: Generating unit bids

	Block 1		Block 2		Block 3		Block 4	
Unit	Size [MW]	Price [$/MWh]	Size [MW]	Price [$/MWh]	Size [MW]	Price [$/MWh]	Size [MW]	Price [$/MWh]
1,2	15.8	29.58	0.2	30.42	3.8	42.82	0.2	43.28
3,4	15.2	11.46	22.8	11.96	22.8	13.89	15.2	15.97
5,6	15.8	29.58	0.2	30.42	3.8	42.82	0.2	43.28
7,8	15.2	11.46	22.8	11.96	22.8	13.89	15.2	15.97
9-11	25.0	18.60	25.0	20.03	30.0	21.67	20.0	22.72
12-14	68.95	19.20	49.25	20.32	39.4	21.22	39.4	22.13
15-19	2.4	23.41	3.6	23.78	3.6	26.84	2.4	30.40
20,21	54.25	9.92	38.75	10.25	31.0	10.68	31.0	11.26
22,23	100.0	5.31	100.0	5.38	120.0	5.53	80.0	5.66
24-29	50.0	0						
30,31	54.25	9.92	38.75	10.25	31.0	10.68	31.0	11.26
32	140	10.08	87.5	10.66	52.5	11.09	70.0	11.72

C.3 Data for Consumers

Every demand is modeled through three consumption blocks. The size and price of each block of each demand for the peak load hour are shown in Table C.5. It is considered that the price bid by each demand corresponds to its utility.

Table C.5: Demand bids for the peak load hour

Node	Block 1		Block 2		Block 3	
	Size [MW]	Price [$/MWh]	Size [MW]	Price [$/MWh]	Size [MW]	Price [$/MWh]
1	104.40	22.80	7.20	20.73	7.20	18.65
2	93.68	22.81	6.46	20.74	6.46	18.66
3	174.00	22.56	12.00	20.51	12.00	18.46
4	71.54	23.33	4.93	21.21	4.93	19.08
5	68.64	23.20	4.73	21.09	4.73	18.98
6	131.48	23.30	9.06	21.18	9.06	19.06
7	120.84	23.83	8.33	21.67	8.33	19.50
8	165.30	24.15	11.40	21.96	11.40	19.76
9	169.18	22.84	11.66	20.76	11.66	18.68
10	188.50	22.94	13.00	20.85	13.00	18.76
13	256.18	22.42	17.66	20.38	17.66	18.34
14	187.54	22.45	12.93	20.41	12.93	18.36
15	306.44	21.69	21.13	19.72	21.13	17.74
16	96.68	21.74	6.66	19.77	6.66	17.79
18	321.90	21.24	22.20	19.31	22.20	17.37
19	174.98	21.83	12.06	19.85	12.06	17.86
20	123.74	21.74	8.53	19.76	8.53	17.79

The minimum power requirement of each demand for the peak load hour is shown in Table C.6. Table C.7 provides the weekly peak loads as a percentage of the annual peak. Table C.8 gives a daily peak load cycle, as a percentage of the weekly peak load. Table C.9 shows the hourly load as a percentage of the daily peak load. In this table, “Wkdy” refers to weekday and “Wknd” to weekend.

For example, if we need to obtain the equilibrium in week 11 (71.5 %), on Friday (94 %), in hour 8-9 am (95 %), then size bids of the demands for this hour are obtained multiplying size bid values of Table C.5 by $0.715 \times 0.94 \times 0.95$, and the same for the minimum demand requirements in Table C.6.

Table C.6: Minimum demand

Node	Minimum demand [MW]	Node	Minimum demand [MW]
1	97.20	10	175.50
2	87.22	13	238.52
3	162.00	14	174.61
4	66.61	15	285.31
5	63.91	16	90.02
6	122.42	18	299.70
7	112.51	19	162.92
8	153.90	20	115.21
9	157.52		

Table C.7: Weekly peak load as a percentage of annual peak

Week	Peak load [%]	Week	Peak load [%]	Week	Peak load [%]
1	86.2	19	87.0	37	78.0
2	90.0	20	88.0	38	69.5
3	87.8	21	85.6	39	72.4
4	83.4	22	81.1	40	72.4
5	88.0	23	90.0	41	74.3
6	84.1	24	88.7	42	74.4
7	83.2	25	89.6	43	80.0
8	80.6	26	86.1	44	88.1
9	74.0	27	75.5	45	88.5
10	73.7	28	81.6	46	90.9
11	71.5	29	80.1	47	94.0
12	72.7	30	88.0	48	89.0
13	70.4	31	72.2	49	94.2
14	75.0	32	77.6	50	97.0
15	72.1	33	80.0	51	100.0
16	80.0	34	72.9	52	95.2
17	75.4	35	72.6		
18	83.7	36	70.5		

Table C.8: Daily load as a percentage of weekly peak

Day	Peak load [%]
Monday	93
Tuesday	100
Wednesday	98
Thursday	96
Friday	94
Saturday	77
Sunday	75

Table C.9: Hourly peak load as a percentage of daily peak

Hour	Winter weeks 1-8 and 44-52		Summer weeks 18-30		Spring / fall weeks 9-17 and 31-43	
	Wkdy	Wknd	Wkdy	Wknd	Wkdy	Wknd
12-1am	67	78	64	74	63	75
1-2	63	72	60	70	62	73
2-3	60	68	58	66	60	69
3-4	59	66	56	65	58	66
4-5	59	64	56	64	59	65
5-6	60	65	58	62	65	65
6-7	74	66	64	62	72	68
7-8	86	70	76	66	85	74
8-9	95	80	87	81	95	83
9-10	96	88	95	86	99	89
10-11	96	90	99	91	100	92
11-noon	95	91	100	93	99	94
noon-1pm	95	90	99	93	93	91
1-2	95	88	100	92	92	90
2-3	93	87	100	91	90	90
3-4	94	87	97	91	88	86
4-5	99	91	96	92	90	85
5-6	100	100	96	94	92	88
6-7	100	99	93	95	96	92
7-8	96	97	92	95	98	100
8-9	91	94	92	100	96	97
9-10	83	92	93	93	90	95
10-11	73	87	87	88	80	90
11-12	63	81	72	80	70	85

C.4 Network Data

Figure C.1 depicts the IEEE 24-node Reliability Test System. The transmission lines include two voltage levels, 138 and 230 kV as can be seen in Figure C.1. There are 230 / 138 kV transformers at nodes 11-9, 11-10, 12-9, 12-10 and 24-3.

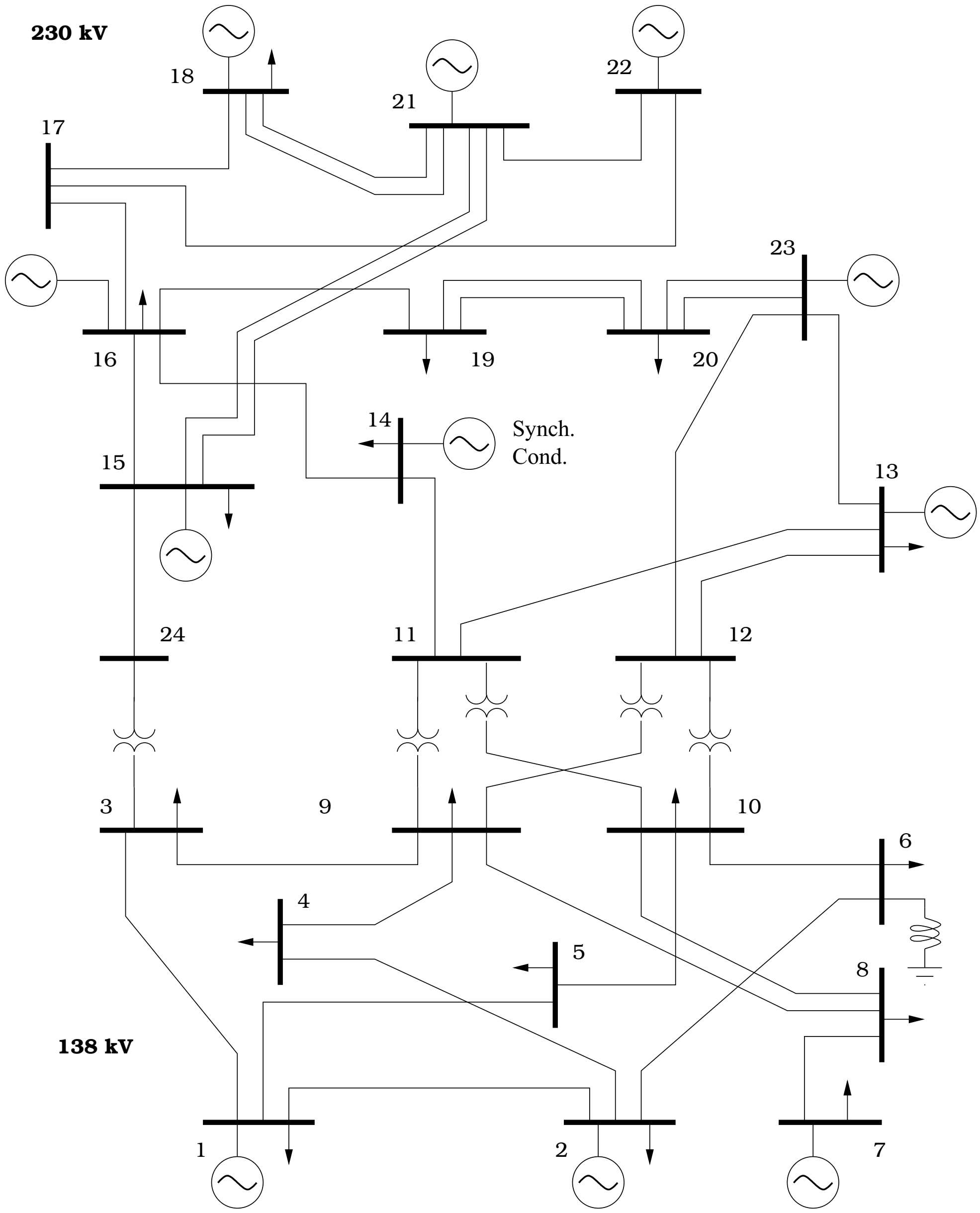

Figure C.1: IEEE 24-node Reliability Test System

Impedance and capacity data for lines and transformers are given in Table C.10. All per unit quantities are on a 100 MVA base, and 138 kV for 138 kV lines or 230 kV for 230 kV lines.

Table C.10: Transmission line data

From node	To node	R [pu]	X [pu]	B [pu]	Capacity [MW]	Equipment
1	2	0.0026	0.0139	0.4611	175	138 kV cable
1	3	0.0546	0.2112	0.0572	175	138 kV line
1	5	0.0218	0.0845	0.0229	175	138 kV line
2	4	0.0328	0.1267	0.0343	175	138 kV line
2	6	0.0497	0.1920	0.0520	175	138 kV line
3	9	0.0308	0.1190	0.0322	175	138 kV line
3	24	0.0023	0.0839	—	400	Transformer
4	9	0.0268	0.1037	0.0281	175	138 kV line
5	10	0.0228	0.0883	0.0239	175	138 kV line
6	10	0.0139	0.0605	2.4590	175	138 kV cable
7	8	0.0159	0.0614	0.0166	175	138 kV line
8	9	0.0427	0.1651	0.0447	175	138 kV line
8	10	0.0427	0.1651	0.0447	175	138 kV line
9	11	0.0023	0.0839	—	175	Transformer
9	12	0.0023	0.0839	—	175	Transformer
10	11	0.0023	0.0839	—	400	Transformer
10	12	0.0023	0.0839	—	400	Transformer
11	13	0.0061	0.0476	0.0999	400	230 kV line
11	14	0.0054	0.0418	0.0879	500	230 kV line
12	13	0.0061	0.0476	0.0999	500	230 kV line
12	23	0.0124	0.0966	0.2030	500	230 kV line
13	23	0.0111	0.0865	0.1818	500	230 kV line
14	16	0.0050	0.0389	0.0818	500	230 kV line
15	16	0.0022	0.0173	0.0364	500	230 kV line
15	21	0.0063	0.0490	0.1030	400	230 kV line
15	21	0.0063	0.0490	0.1030	400	230 kV line
15	24	0.0067	0.0519	0.1091	500	230 kV line
16	17	0.0033	0.0259	0.0545	500	230 kV line
16	19	0.0030	0.0231	0.0485	500	230 kV line
17	18	0.0018	0.0144	0.0303	500	230 kV line
17	22	0.0135	0.1053	0.2212	500	230 kV line
18	21	0.0033	0.0259	0.0545	500	230 kV line
18	21	0.0033	0.0259	0.0545	500	230 kV line
19	20	0.0051	0.0396	0.0833	500	230 kV line
19	20	0.0051	0.0396	0.0833	500	230 kV line
20	23	0.0028	0.0216	0.0455	500	230 kV line
20	23	0.0028	0.0216	0.0455	500	230 kV line
21	22	0.0087	0.0678	0.1424	500	230 kV line

Appendix D

Results: IEEE 24-Node Reliability Test System

This appendix collects additional results pertaining to the case studies analyzed in Chapter 5.

D.1 Single-Period Case

This section includes additional information on cases 2 and 3, presented in Subsection 5.2 of Chapter 5. Note that these cases include Minimum Profit Conditions (MPC).

D.1.1 Case 2

Table D.1 provides results for the generating units, Table D.2 for the demands and Table D.3 provides locational marginal prices of case 2. Note that we provide profit values and demand cost values once the corresponding infeasibility costs have been subtracted and added, respectively.

D.1.2 Case 3

Table D.4, Table D.5 and Table D.6 provide results for the generating units, for the demands and locational marginal prices, respectively, of case 3. Note that the profit values and the demand cost values provided in these tables include infeasibility costs.

D.2 Multi-Period Case

This section includes additional results on the multi-period equilibrium with and without minimum profit conditions stated in Subsection 5.3 in Chapter

5. Results pertaining to the succession of single-period equilibria are also provided.

D.2.1 No Minimum Profit Condition Case

Tables D.7 and D.8 collect locational marginal prices for each node of the system by time period for the multi-period equilibrium without including minimum profit constraints. Tables D.9 and D.10 provide the power output of each generating unit by time period, and Tables D.11 and D.12 provide the profit of each generating unit by time period.

D.2.2 Succession of Single-Period Equilibria

Table D.13 provides results pertaining to the generating units for the succession of single-period equilibria. Table D.14 includes additional information by time period.

D.2.3 Minimum Profit Condition Case

Tables D.15 and D.16 provide locational marginal prices by node and by time period for the multi-period equilibrium with minimum profit constraints. Tables D.17 and D.18 collect the power output of each generating unit by time period, and Tables D.19 and D.20 provide the profit of each generating unit by time period.

Table D.1: Results for the generating units. Single-period equilibrium with MPC. Case 2

Unit	Power output [MW]	Revenue [$/h]	Profit [$/h]
1, 2	0.00	0.00	0.00
3, 4	76.00	1629.40	622.29
5, 6	0.00	0.00	0.00
7, 8	76.00	1639.41	632.29
9	79.33	1719.08	116.92
10, 11	50.00	1083.50	117.22
12-14	118.20	2524.59	198.74
15-19	0.00	0.00	0.00
20	155.00	3156.10	1538.97
21	155.00	3169.26	1552.13
22	400.00	7941.34	5751.72
23	400.00	7908.30	5718.68
24-29	50.00	959.90	959.37
30, 31	155.00	3155.95	1538.82
32	350.00	7126.34	3376.06
Total	2907.93	59370.15	30228.23

Table D.2: Results for the demands. Single-period equilibrium with MPC. Case 2

Demand	Power consumed [MW]	Demand cost [$/h]	Demand	Power consumed [MW]	Demand cost [$/h]
1	111.60	2388.85	10	188.50	4112.18
2	93.68	2021.79	13	260.47	5562.17
3	186.00	3933.70	14	187.54	4007.21
4	71.54	1583.69	15	327.57	6659.47
5	68.64	1510.10	16	103.34	2109.09
6	131.48	2933.67	18	344.10	6823.24
7	129.17	2800.50	19	187.04	3834.66
8	165.30	3698.77	20	132.27	2700.01
9	169.18	3677.49	Total	2857.42	60357.20

Table D.3: Locational marginal prices. Single-period equilibrium with MPC. Case 2

Node	Locational marginal price [$/MWh]	Node	Locational marginal price [$/MWh]
1	21.44	13	21.36
2	21.57	14	21.36
3	21.18	15	20.36
4	22.13	16	20.45
5	21.99	17	20.03
6	22.30	18	19.85
7	21.67	19	20.53
8	22.37	20	20.45
9	21.73	21	19.77
10	21.80	22	19.20
11	21.63	23	20.36
12	21.63	24	20.97

Table D.4: Results for the generating units. Single-period equilibrium with MPC. Case 3

Unit	Power output [MW]	Revenue [$/h]	Profit [$/h]
1, 2	0.00	0.00	0.00
3, 4	76.00	1690.91	684.54
5, 6	0.00	0.00	0.00
7, 8	76.00	1701.29	694.92
9	75.63	1679.51	158.28
10	50.00	1110.33	144.52
11	80.00	1776.53	160.62
12, 13	118.20	2624.60	299.91
14	0.00	0.00	0.00
15-19	0.00	0.00	0.00
20	155.00	3281.12	1665.51
21	155.00	3294.80	1679.19
22	400.00	8255.91	6070.19
23	400.00	8221.57	6035.85
24-29	50.00	997.93	997.89
30, 31	155.00	3280.96	1665.36
32	350.00	7408.63	3661.87
Total	2816.03	59611.50	32252.68

Table D.5: Results for the demands. Single-period equilibrium with MPC. Case 3

Demand	Power consumed [MW]	Demand cost [$/h]	Demand	Power consumed [MW]	Demand cost [$/h]
1	104.40	2322.87	10	188.50	4265.45
2	93.68	2097.15	13	256.18	5688.62
3	174.00	3831.66	14	187.54	4164.01
4	71.54	1642.74	15	306.44	6487.02
5	68.64	1566.40	16	96.68	2055.19
6	131.48	3043.05	18	321.90	6644.20
7	129.17	2864.46	19	174.98	3735.49
8	165.30	3829.67	20	123.74	2630.50
9	169.18	3821.41	Total	2763.35	60689.68

Table D.6: Locational marginal prices. Single-period equilibrium with MPC. Case 3

Node	Locational marginal price [$/MWh]	Node	Locational marginal price [$/MWh]
1	22.25	13	22.20
2	22.39	14	22.20
3	22.02	15	21.17
4	22.96	16	21.26
5	22.82	17	20.82
6	23.14	18	20.64
7	22.21	19	21.35
8	23.17	20	21.26
9	22.59	21	20.55
10	22.63	22	19.96
11	22.49	23	21.17
12	22.45	24	21.80

Table D.7: Locational marginal prices by time periods [$/MWh]. Multi-period equilibrium without MPC

Node	Hour											
	1	2	3	4	5	6	7	8	9	10	11	12
1	12.27	11.96	11.83	11.77	11.77	11.83	13.89	20.64	21.01	21.15	21.15	21.01
2	12.27	11.96	11.96	11.96	11.96	11.96	13.89	20.66	21.01	21.15	21.15	21.01
3	11.66	11.35	11.06	11.04	11.04	11.06	13.20	20.18	20.55	20.68	20.68	20.55
4	12.55	12.16	11.84	11.69	11.69	11.84	14.21	21.14	21.49	21.63	21.63	21.49
5	12.55	12.22	11.90	11.75	11.75	11.90	14.21	21.11	21.49	21.63	21.63	21.49
6	12.51	12.19	11.87	11.72	11.72	11.87	14.17	21.18	21.49	21.63	21.63	21.49
7	13.43	13.08	12.73	12.57	12.57	12.73	15.20	21.62	21.67	21.67	21.67	21.67
8	13.13	12.78	12.45	12.29	12.29	12.45	14.86	22.11	22.17	22.17	22.17	22.17
9	12.27	11.89	11.58	11.43	11.43	11.58	13.89	20.66	21.01	21.15	21.15	21.01
10	12.27	11.95	11.63	11.49	11.49	11.63	13.89	20.76	21.07	21.20	21.20	21.07
11	12.18	11.86	11.55	11.40	11.40	11.55	13.79	20.61	20.92	21.05	21.05	20.92
12	12.18	11.86	11.55	11.40	11.40	11.55	13.79	20.61	20.92	21.05	21.05	20.92
13	12.04	11.73	11.42	11.28	11.28	11.42	13.64	20.39	20.68	20.82	20.82	20.68
14	11.77	11.47	11.17	11.03	11.03	11.17	13.33	20.16	20.68	20.81	20.81	20.68
15	11.14	10.84	10.56	10.54	10.54	10.56	12.61	19.28	19.72	19.75	19.75	19.72
16	11.26	10.97	10.68	10.66	10.66	10.68	12.75	19.49	19.94	19.97	19.97	19.94
17	11.01	10.61	10.33	10.31	10.31	10.33	12.46	19.05	19.49	19.52	19.52	19.49
18	10.89	10.49	10.22	10.20	10.20	10.22	12.33	18.85	19.28	19.31	19.31	19.28
19	11.39	11.09	10.80	10.78	10.78	10.80	12.90	19.71	20.17	20.20	20.20	20.17
20	11.52	11.22	10.92	10.90	10.90	10.92	13.04	19.49	19.94	19.97	19.97	19.94
21	10.77	10.49	10.21	10.19	10.19	10.21	12.19	18.64	19.07	19.10	19.10	19.07
22	10.41	10.14	9.88	9.86	9.86	9.88	11.79	18.02	18.44	18.47	18.47	18.44
23	11.39	11.09	10.80	10.78	10.78	10.80	12.90	19.27	19.72	19.75	19.75	19.72
24	11.52	11.22	10.93	10.91	10.91	10.93	13.04	19.94	20.40	20.43	20.43	20.40

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Table D.8: Locational marginal prices by time periods [$/MWh]. Multi-period equilibrium without MPC (continued)

Node	Hour											
	13	14	15	16	17	18	19	20	21	22	23	24
1	21.01	21.01	20.77	20.83	21.67	21.67	22.72	21.56	20.71	18.76	18.69	12.88
2	21.01	21.01	20.77	20.83	21.67	21.67	22.72	21.56	20.71	19.07	18.69	13.09
3	20.55	20.55	20.30	20.36	21.19	21.19	22.21	21.08	20.25	18.58	17.56	11.79
4	21.49	21.49	21.24	21.30	22.16	22.16	23.24	22.05	21.19	19.50	19.11	12.83
5	21.49	21.49	21.24	21.30	22.16	22.16	23.24	22.05	21.18	19.19	19.11	12.83
6	21.49	21.49	21.24	21.30	22.11	22.11	23.18	22.05	21.19	19.50	19.06	12.80
7	21.67	21.67	21.67	21.67	21.67	21.67	22.72	22.55	21.67	20.92	20.45	14.37
8	22.17	22.17	22.17	22.17	22.17	22.17	23.24	23.07	22.17	20.46	20.00	14.04
9	21.01	21.01	20.77	20.83	21.67	21.67	22.72	21.56	20.71	19.07	18.69	12.54
10	21.07	21.07	20.82	20.88	21.67	21.67	22.72	21.61	20.77	19.12	18.69	12.54
11	20.92	20.92	20.67	20.73	21.52	21.52	22.56	21.46	20.62	18.98	18.55	12.45
12	20.92	20.92	20.67	20.73	21.52	21.52	22.56	21.46	20.62	18.98	18.55	12.45
13	20.68	20.68	20.44	20.50	21.28	21.28	22.31	21.22	20.39	18.77	18.35	12.32
14	20.68	20.68	20.44	20.50	21.27	21.27	22.31	21.22	20.39	18.56	17.94	12.04
15	19.72	19.72	19.49	19.54	20.34	20.34	21.32	20.13	19.34	17.75	16.77	11.26
16	19.94	19.94	19.70	19.76	20.56	20.56	21.56	20.36	19.56	17.95	16.96	11.39
17	19.49	19.49	19.26	19.32	20.10	20.10	21.08	19.90	19.12	17.54	16.58	11.01
18	19.28	19.28	19.05	19.11	19.88	19.88	20.85	19.69	18.91	17.35	16.40	10.89
19	20.17	20.17	19.93	19.99	20.80	20.80	21.81	20.59	19.78	18.15	17.15	11.52
20	19.94	19.94	19.71	19.76	20.56	20.57	21.56	20.36	19.56	17.95	17.35	11.65
21	19.07	19.07	18.84	18.90	19.66	19.66	20.62	19.47	18.70	17.16	16.22	10.89
22	18.44	18.44	18.22	18.27	19.01	19.01	19.94	18.82	18.09	16.59	15.68	10.53
23	19.72	19.72	19.48	19.54	20.33	20.33	21.32	20.13	19.34	17.75	17.15	11.51
24	20.40	20.40	20.16	20.22	21.04	21.04	22.06	20.83	20.01	18.36	17.35	11.65

Table D.9: Power output of the units by time periods [MW]. Multi-period equilibrium without MPC

Unit	Hour											
	1	2	3	4	5	6	7	8	9	10	11	12
1,2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	38.0	19.9	15.2	15.2	15.2	15.2	45.0	76.0	76.0	76.0	76.0	76.0
4	38.0	15.2	15.2	15.2	15.2	15.2	60.8	76.0	76.0	76.0	76.0	76.0
5,6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	38.0	27.8	38.0	38.0	38.0	38.0	38.0	76.0	76.0	76.0	76.0	76.0
8	38.0	38.0	24.7	19.5	19.5	24.7	48.4	76.0	76.0	76.0	76.0	76.0
9	0.0	0.0	0.0	0.0	0.0	0.0	25.0	50.0	50.0	50.0	50.0	77.5
10	25.0	25.0	25.0	25.0	25.0	25.0	25.0	50.0	77.5	50.4	50.4	50.0
11	0.0	0.0	0.0	0.0	0.0	0.0	25.0	50.0	50.0	80.0	80.0	50.0
12,13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	118.2	118.2	118.2	118.2
15-19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	124.0	124.0	93.0	93.0	93.0	93.0	155.0	155.0	155.0	155.0	155.0	155.0
21	134.1	124.0	119.2	93.0	93.0	119.2	155.0	155.0	155.0	155.0	155.0	155.0
22,23	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0
24-29	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0
30,31	155.0	124.0	124.0	124.0	124.0	124.0	155.0	155.0	155.0	155.0	155.0	155.0
32	280.0	279.6	227.5	227.5	227.5	227.5	350.0	350.0	350.0	350.0	350.0	350.0

continued on next table

Table D.10: Power output of the units by time periods [MW]. Multi-period equilibrium without MPC (continued)

Unit	Hour											
	13	14	15	16	17	18	19	20	21	22	23	24
1,2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	0.0
4	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	0.0	0.0
5,6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	0.0	0.0
8	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	38.0
9	77.5	77.5	50.0	50.0	53.7	80.0	0.0	0.0	0.0	0.0	0.0	0.0
10	50.0	50.0	50.0	50.0	80.0	80.0	88.6	80.0	80.0	50.0	0.0	0.0
11	50.0	50.0	71.6	74.5	80.0	80.0	100.0	80.0	73.3	50.0	50.0	0.0
12,13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	118.2	118.2	118.2	118.2	157.6	157.6	197.0	124.3	118.2	0.0	0.0	0.0
15-19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	130.8
21	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0
22,23	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0
24-29	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0
30,31	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0
32	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	280.0

Table D.11: Profit of the units by time periods [$/h]. Multi-period equilibrium without MPC

Unit	Hour											
	1	2	3	4	5	6	7	8	9	10	11	12
1,2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	11.8	0.1	-1.9	-2.8	-2.8	-1.9	73.4	554.9	583.2	593.5	593.5	583.2
4	11.8	0.1	-1.9	-2.8	-2.8	-1.9	73.4	554.9	583.2	593.5	593.5	583.2
5, 6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	11.7	0.1	0.1	0.1	0.1	0.1	73.4	556.6	583.2	593.4	593.4	583.2
8	11.7	0.1	0.1	0.1	0.1	0.1	73.4	556.6	583.2	593.4	593.4	583.2
9	0.0	0.0	0.0	0.0	0.0	0.0	-668.4	106.6	109.3	109.3	109.3	109.3
10	-712.8	-146.6	-155.1	-159.2	-159.2	-155.1	-93.4	106.6	109.3	109.3	109.3	109.3
11	0.0	0.0	0.0	0.0	0.0	0.0	-668.4	106.6	109.3	109.3	109.3	109.3
12,13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-913.7	120.9	120.9	105.2
15-19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	108.2	72.1	40.7	38.9	38.9	40.7	332.8	1366.2	1434.9	1439.7	1439.7	1434.9
21	-188.4	87.1	51.6	49.8	49.8	51.6	354.6	1399.6	1469.0	1473.9	1473.9	1469.0
22	2169.5	2011.0	1901.6	1894.2	1894.2	1901.6	2746.1	5353.5	5526.7	5539.0	5539.0	5526.7
23	2121.4	2008.9	1899.6	1892.1	1892.1	1899.6	2691.6	5270.2	5441.4	5453.6	5453.6	5441.4
24-29	520.6	507.0	493.8	492.9	492.9	493.8	589.5	901.2	921.9	923.3	923.3	921.9
30,31	143.3	102.5	66.6	64.2	64.2	66.6	377.0	1365.9	1434.5	1439.4	1439.4	1434.5
32	242.3	159.0	93.3	88.8	88.8	93.3	746.7	2979.6	3134.6	3145.6	3145.6	3134.6

continued on next table

Table D.12: Profit of the units by time periods [$/h]. Multi-period equilibrium without MPC (continued)

Unit	Hour											
	13	14	15	16	17	18	19	20	21	22	23	24
1,2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	583.2	583.2	564.4	569.0	633.1	633.2	713.0	624.6	560.4	411.8	406.4	0.0
4	583.2	583.2	564.4	569.0	633.1	633.2	713.0	624.6	560.4	411.8	0.0	0.0
5,6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	583.2	583.2	564.3	569.0	633.1	633.1	712.9	624.6	560.4	435.3	0.0	0.0
8	583.2	583.2	564.3	569.0	633.1	633.1	712.9	624.6	560.4	435.3	406.3	43.1
9	109.3	109.3	109.3	109.3	109.3	109.3	0.0	0.0	0.0	0.0	0.0	0.0
10	109.3	109.3	109.3	109.3	109.3	109.3	193.3	180.0	109.3	71.9	0.0	0.0
11	109.3	109.3	109.3	109.3	109.3	109.3	193.3	180.0	109.3	71.9	48.4	0.0
12,13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	105.2	105.2	76.3	83.4	177.5	177.5	346.9	168.6	70.2	0.0	0.0	0.0
15-19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	1434.9	1434.9	1398.7	1407.6	1530.3	1530.5	1683.2	1499.1	1376.7	1129.2	978.1	123.6
21	1469.0	1469.0	1432.4	1441.4	1565.5	1565.6	1720.1	1534.0	1410.2	1159.9	1007.1	143.0
22	5526.7	5526.7	5435.5	5457.9	5767.5	5767.9	6153.2	5688.8	5380.0	4755.5	4374.2	2171.7
23	5441.4	5441.4	5351.3	5373.4	5679.6	5679.9	6061.1	5601.8	5296.4	4678.7	4301.7	2169.5
24-29	921.9	921.9	911.0	913.6	950.6	950.7	996.8	941.2	904.3	829.7	784.1	526.4
30,31	1434.5	1434.5	1398.4	1407.2	1529.9	1530.1	1682.8	1498.8	1376.4	1128.9	1036.9	163.0
32	3134.6	3134.6	3053.0	3073.0	3350.1	3350.4	3695.2	3279.7	3003.3	2444.5	2236.7	277.9

Table D.13: Results for the generating units. Succession of single-period equilibria

Unit	Total energy [MWh]	Profit [k$]	Unit	Total energy [MWh]	Profit [k$]
1, 2	0.00	0.00	14	1559.38	-0.22
3	1413.60	8.42	15-19	0.00	0.00
4	1455.20	8.42	20	3405.18	22.65
5, 6	0.00	0.00	21	3534.00	22.67
7	1548.41	8.62	22	9600.00	102.52
8	1493.21	8.62	23	9600.00	101.66
9	759.75	0.86	24-29	1200.00	18.73
10	1165.96	0.86	30	3539.69	23.07
11	791.99	0.88	31	3534.00	23.07
12	1233.85	-0.22	32	5786.95	35.89
13	1556.30	-0.22	Total	57977.47	461.20

Table D.14: Results by time periods. Succession of single-period equilibria

Hour	Gen. unit profit [k$/h]	Gen. unit revenue [k$/h]	Demand cost [k$/h]	Minimum LMP [$/MWh]	Maximum LMP [$/MWh]
1	8.17	23.77	24.72	10.59	14.19
2	7.63	21.47	22.32	10.20	13.56
3	7.37	20.15	20.94	10.05	13.15
4	7.31	19.75	20.54	10.01	13.14
5	7.31	19.75	20.54	10.01	13.14
6	7.37	20.15	20.94	10.05	13.15
7	15.03	35.32	36.23	14.36	18.03
8	20.70	45.45	46.34	16.87	20.26
9	26.01	54.29	55.17	18.50	21.96
10	27.20	54.68	55.51	18.58	21.96
11	27.20	54.68	55.51	18.58	21.96
12	27.03	54.29	55.17	18.50	21.96
13	27.03	54.29	55.17	18.50	21.96
14	27.03	54.29	55.17	18.50	21.96
15	26.97	53.34	54.21	18.48	21.92
16	26.98	53.99	54.86	18.48	21.92
17	27.37	55.47	56.29	18.63	21.96
18	27.40	55.52	56.34	18.65	21.96
19	27.40	55.52	56.34	18.65	21.96
20	27.20	54.68	55.51	18.58	21.96
21	26.97	52.33	53.17	18.48	21.92
22	22.69	43.75	44.65	16.83	20.26
23	16.64	34.70	35.68	14.26	18.07
24	7.94	21.79	22.64	10.36	13.76

Table D.15: Locational marginal prices by time periods [$/MWh]. Multi-period equilibrium with MPC

Node	Hour											
	1	2	3	4	5	6	7	8	9	10	11	12
1	12.40	11.96	12.00	11.96	11.96	12.00	19.00	20.57	21.72	22.54	21.56	21.11
2	12.40	11.96	11.96	11.96	11.96	11.96	19.32	20.64	21.72	22.54	21.56	21.11
3	11.79	11.37	11.27	11.24	11.24	11.27	18.58	20.12	21.24	22.04	21.08	20.64
4	12.69	12.23	12.21	12.17	12.17	12.21	19.76	21.12	22.22	23.05	22.05	21.60
5	12.69	12.23	12.27	12.23	12.23	12.27	19.44	21.04	22.22	23.05	22.05	21.60
6	12.65	12.20	12.24	12.20	12.20	12.24	19.95	21.12	22.22	23.00	22.05	21.60
7	14.20	13.70	13.13	13.09	13.09	13.13	22.28	22.65	23.84	24.67	22.55	22.09
8	13.89	13.39	12.84	12.80	12.80	12.84	21.78	22.15	23.30	24.12	23.07	22.60
9	12.40	11.96	11.94	11.90	11.90	11.94	19.46	20.64	21.72	22.54	21.56	21.12
10	12.40	11.96	11.99	11.96	11.96	11.99	19.55	20.70	21.78	22.54	21.61	21.17
11	12.31	11.87	11.91	11.87	11.87	11.91	19.41	20.55	21.62	22.38	21.46	21.02
12	12.31	11.87	11.91	11.87	11.87	11.91	19.41	20.55	21.62	22.38	21.46	21.02
13	12.18	11.74	11.78	11.74	11.74	11.78	19.19	20.32	21.38	22.13	21.22	20.78
14	11.91	11.48	11.51	11.48	11.48	11.51	18.76	20.10	21.38	22.13	21.22	20.78
15	11.26	10.86	10.76	10.73	10.73	10.76	17.75	19.22	20.29	21.05	20.13	19.72
16	11.39	10.98	10.88	10.85	10.85	10.88	17.95	19.43	20.51	21.29	20.36	19.94
17	11.13	10.62	10.53	10.50	10.50	10.53	17.54	18.99	20.05	20.81	19.90	19.49
18	11.01	10.50	10.41	10.38	10.38	10.41	17.35	18.79	19.84	20.58	19.69	19.28
19	11.52	11.10	11.01	10.98	10.98	11.01	18.15	19.65	20.75	21.53	20.59	20.17
20	11.65	11.23	11.13	11.10	11.10	11.13	18.36	19.43	20.52	21.29	20.36	19.94
21	10.89	10.50	10.41	10.38	10.38	10.41	17.16	18.58	19.62	20.35	19.47	19.07
22	10.53	10.15	10.06	10.04	10.04	10.06	16.59	17.97	18.97	19.68	18.82	18.44
23	11.51	11.10	11.26	11.23	11.23	11.26	18.15	19.21	20.29	21.05	20.13	19.72
24	11.65	11.23	11.13	11.10	11.10	11.13	18.36	19.88	20.99	21.77	20.83	20.40

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Table D.16: Locational marginal prices by time periods [$/MWh]. Multi-period equilibrium with MPC (continued)

Node	Hour											
	13	14	15	16	17	18	19	20	21	22	23	24
1	21.11	21.11	20.77	20.98	22.54	22.72	22.72	21.56	20.71	18.76	13.89	11.96
2	21.11	21.11	20.77	20.98	22.54	22.72	22.72	21.56	20.71	19.07	13.89	11.96
3	20.64	20.64	20.30	20.52	22.04	22.21	22.21	21.08	20.25	18.58	13.05	11.35
4	21.60	21.60	21.24	21.46	23.05	23.24	23.24	22.05	21.19	19.50	14.21	12.16
5	21.60	21.60	21.24	21.46	23.05	23.24	23.24	22.05	21.18	19.19	14.21	12.22
6	21.60	21.60	21.24	21.46	23.00	23.18	23.18	22.05	21.19	19.50	14.17	12.19
7	22.09	22.09	21.72	21.95	22.72	22.72	22.72	22.55	21.67	20.92	15.20	13.08
8	22.60	22.60	22.22	22.45	23.24	23.24	23.24	23.07	22.17	20.46	14.86	12.78
9	21.12	21.12	20.77	20.98	22.54	22.72	22.72	21.56	20.71	19.07	13.89	11.89
10	21.17	21.17	20.82	21.04	22.54	22.72	22.72	21.61	20.77	19.12	13.89	11.95
11	21.02	21.02	20.67	20.89	22.38	22.56	22.56	21.46	20.62	18.98	13.79	11.86
12	21.02	21.02	20.67	20.89	22.38	22.56	22.56	21.46	20.62	18.98	13.79	11.86
13	20.78	20.78	20.44	20.65	22.13	22.31	22.31	21.22	20.39	18.77	13.64	11.73
14	20.78	20.78	20.44	20.65	22.13	22.31	22.31	21.22	20.39	18.56	13.33	11.47
15	19.72	19.72	19.49	19.69	21.15	21.32	21.32	20.13	19.34	17.75	12.47	10.84
16	19.94	19.94	19.70	19.91	21.39	21.56	21.56	20.36	19.56	17.95	12.61	10.97
17	19.49	19.49	19.26	19.46	20.91	21.08	21.08	19.90	19.12	17.54	12.32	10.61
18	19.28	19.28	19.05	19.25	20.68	20.85	20.85	19.69	18.91	17.35	12.19	10.49
19	20.17	20.17	19.93	20.14	21.63	21.81	21.81	20.59	19.78	18.15	12.75	11.09
20	19.94	19.94	19.71	19.91	21.39	21.56	21.56	20.36	19.56	17.95	12.90	11.22
21	19.07	19.07	18.84	19.04	20.45	20.62	20.62	19.47	18.70	17.16	12.06	10.49
22	18.44	18.44	18.22	18.41	19.78	19.94	19.94	18.82	18.09	16.59	11.66	10.14
23	19.72	19.72	19.48	19.69	21.15	21.32	21.32	20.13	19.34	17.75	12.75	11.09
24	20.40	20.40	20.16	20.37	21.88	22.06	22.06	20.83	20.01	18.36	12.90	11.22

Table D.17: Power output of the units by time periods [MW]. Multi-period equilibrium with MPC

Unit	Hour											
	1	2	3	4	5	6	7	8	9	10	11	12
1,2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	38.0	24.5	38.0	38.0	38.0	38.0	76.0	76.0	76.0	76.0	76.0	76.0
4	38.0	38.0	38.0	26.3	26.3	38.0	76.0	76.0	76.0	76.0	76.0	76.0
5,6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	38.0	27.8	38.0	23.6	23.6	38.0	76.0	76.0	76.0	76.0	76.0	76.0
8	38.0	38.0	38.0	38.0	38.0	38.0	76.0	76.0	76.0	76.0	76.0	76.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	80.0	80.0
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	80.0	100.0	100.0	80.0	80.0
12,13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.0	0.0	0.0	0.0	0.0	0.0	0.0	83.2	157.6	180.1	124.3	118.2
15-19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	134.2	124.0	124.0	124.0	124.0	124.0	155.0	155.0	155.0	155.0	155.0	155.0
21	155.0	124.0	124.0	124.0	124.0	124.0	155.0	155.0	155.0	155.0	155.0	155.0
22,23	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0
24-29	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0
30	155.0	124.0	0.0	0.0	0.0	0.0	0.0	155.0	155.0	155.0	155.0	155.0
31	155.0	124.0	127.6	124.0	124.0	127.6	155.0	155.0	155.0	155.0	155.0	155.0
32	280.0	280.0	280.0	280.0	280.0	280.0	350.0	350.0	350.0	350.0	350.0	350.0

continued on next table

Table D.18: Power output of the units by time periods [MW]. Multi-period equilibrium with MPC (continued)

Unit	Hour											
	13	14	15	16	17	18	19	20	21	22	23	24
1,2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	60.8	19.9
4	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	43.0	15.2
5,6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	38.0	27.8
8	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	48.2	38.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	80.0	80.0	80.0	80.0	100.0	100.0	100.0	80.0	80.0	50.0	25.0	25.0
11	80.0	80.0	80.0	80.0	81.1	88.6	88.6	80.0	73.3	50.0	25.0	25.0
12,13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	118.2	118.2	118.2	118.2	182.6	197.0	197.0	124.3	118.2	0.0	0.0	0.0
15-19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	124.0
21	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	124.0
22,23	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0
24-29	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0
30	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	124.0
31	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	124.0
32	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	250.1

Table D.19: Profit of the units by time periods [$/h]. Multi-period equilibrium with MPC

Unit	Hour											
	1	2	3	4	5	6	7	8	9	10	11	12
1,2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3,4	17.0	0.1	1.4	0.1	0.1	1.4	430.4	549.9	637.1	699.2	624.6	590.9
5,6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7,8	16.9	0.1	0.3	0.1	0.1	0.3	454.2	555.1	637.1	699.2	624.6	590.9
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-395.0	142.9
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-387.1	305.0	388.4	180.0	142.9
12,13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-956.7	194.1	312.0	168.6	116.9
15-19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	123.6	73.6	62.1	58.2	58.2	62.1	1129.2	1356.7	1522.9	1641.1	1499.1	1434.9
21	-169.0	88.6	77.0	73.1	73.1	77.0	1159.9	1390.0	1558.0	1677.6	1534.0	1469.0
22	2218.1	2015.8	1980.0	1967.9	1967.9	1980.0	4755.5	5329.5	5748.8	6047.1	5688.8	5526.7
23	2169.5	2013.7	1977.9	1965.8	1965.8	1977.9	4678.7	5246.4	5661.1	5956.1	5601.8	5441.4
24-29	526.4	507.5	503.2	501.8	501.8	503.2	829.7	898.3	948.4	984.1	941.2	921.9
30	163.0	104.1	0.0	0.0	0.0	0.0	0.0	1044.3	1522.5	1640.7	1498.8	1434.5
31	163.0	104.1	123.6	119.5	119.5	123.6	1191.4	1356.3	1522.5	1640.7	1498.8	1434.5
32	277.9	162.6	206.6	197.4	197.4	206.6	2585.7	2958.1	3333.4	3600.3	3279.7	3134.6

continued on next table

Table D.20: Profit of the units by time periods [$/h]. Multi-period equilibrium with MPC (continued)

Unit	Hour											
	13	14	15	16	17	18	19	20	21	22	23	24
1,2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3,4	590.9	590.9	564.4	580.9	699.2	713.0	713.0	624.6	560.4	411.8	73.4	0.1
5,6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7,8	590.9	590.9	564.3	580.9	699.2	712.9	712.9	624.6	560.4	435.3	73.4	0.1
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	142.9	142.9	113.6	131.9	193.3	193.3	193.3	180.0	109.3	71.9	-93.4	-146.6
11	142.9	142.9	113.6	131.9	193.3	193.3	193.3	180.0	109.3	71.9	-93.4	-146.6
12,13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	116.9	116.9	76.3	101.7	312.0	346.9	346.9	168.6	70.2	0.0	0.0	0.0
15-19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	1434.9	1434.9	1398.7	1430.4	1656.8	1683.2	1683.2	1499.1	1376.7	1129.2	310.9	72.1
21	1469.0	1469.0	1432.4	1464.5	1693.4	1720.1	1720.1	1534.0	1410.2	1159.9	332.4	87.1
22	5526.7	5526.7	5435.5	5515.4	6086.6	6153.2	6153.2	5688.8	5380.0	4755.5	2690.7	2011.0
23	5441.4	5441.4	5351.3	5430.3	5995.2	6061.1	6061.1	5601.8	5296.4	4678.7	2636.8	2008.9
24-29	921.9	921.9	911.0	920.5	988.8	996.8	996.8	941.2	904.3	829.7	582.9	507.0
30	1434.5	1434.5	1398.4	1430.0	1656.4	1682.8	1682.8	1498.8	1376.4	1128.9	354.5	102.5
31	1434.5	1434.5	1398.4	1430.0	1656.4	1682.8	1682.8	1498.8	1376.4	1128.9	354.5	102.5
32	3134.6	3134.6	3053.0	3124.5	3635.6	3695.2	3695.2	3279.7	3003.3	2444.5	696.0	159.0

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